STUDIES IN LOGIC

AND THE

FOUNDATIONS OF MATHEMATICS

L. E. J. BROUWER / E. W. BETH / A. HEYTING

EDITORS

Ancient Formal Logic

I. M. BOCHEŃSKI

NORTH-HOLLAND PUBLISHING COMPANY

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I. M. BOCHEŃSKI

Professor of Philosophy
University of Fribourg

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PREFACE

Among the great things which the Atlantic World inherited from Greece, the capacity of understanding pure forms is perhaps one of the greatest and Formal Logic is no small part of this capacity. Unfortunately, Greek Formal Logic is all but unknown to modern men, and what is worse, "an up-to-date picture of the history of ancient Logic is at best a hope for the future". [1] Yet, some of the details of this history can be known. The aim of the present work is to collect some of the data available in the actual state of science and to arrange them in a kind of outline, which would show forth at least some of our indebtedness to Greek Logicians and allow us to see how their results were reached.

This book could not have been written without the direct or indirect help of many scholars to whom the author wishes to express his thanks. To Professor J. Lukasiewicz it owes its general spirit and fundamental ideas. The discussions with the late Fr. J. Salamucha helped considerably in the understanding of several Greek doctrines. The works of Professor H. Scholz, Sir W. D. Ross, Professor F. Solmsen, Dr A. Becker and others were freely used. Dr B. Mates was so kind as to lend the manuscript of his excellent dissertation on Stoic Logic. Both Professor E. W. Beth and Professor K. Dürr supplied valuable information. Fr. I. Thomas was kind enough to read the manuscript and made many suggestions.

It will perhaps be allowed the author to state that when he looks back upon his many years work on the history of logic, he finds that this work would not have been undertaken without the general philosophical background assumed by him. This background is the acknowledgement of the importance of Logic and the high valuation of the so-called "scholastic subtleties". For

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1) Kapp 20.

only one who thinks that reason *has* some uses might become interested in such specculations as are found in the Prior Analytics or in the Stoic Fragments. This belief is, fortunately, also shared by many philosophers belonging to different schools; for them the present book may have meaning. For it attempts to show across many centuries a long line of thinkers who thought and worked in a manner essentially still ours.

CONTENTS

LIST OF SYMBOLS

other than those of the *Principia Mathematica*

"A" "B" "C" "D"		1) class-names (standing for "α", "β" etc.)
"M" "P" "S" "X"		2) functors (standing for "φ", "ψ" etc.)
$\ulcorner F \ulcorner p \urcorner \urcorner$	for	$\ulcorner \ulcorner$p$\urcorner$ is false$\urcorner$
$\ulcorner Tp \ulcorner \urcorner \urcorner$	,,	$\ulcorner \ulcorner$p$\urcorner$ is true$\urcorner$
"V"	means	exclusive alternative (matrix "0110")
"$\rightarrow$"	,,	Diodorean implication (cp. 15. 2)
"⥽"	,,	strict implication (cp. 15. 3)
$\ulcorner \bar{A} \urcorner$	for	$\ulcorner$the contrary of $A\urcorner$ (cp. 7 B)
$\ulcorner SaP \urcorner$	,,	$\ulcorner P$ belongs to all of $S\urcorner$
$\ulcorner SeP \urcorner$	,,	$\ulcorner P$ belongs to no $S\urcorner$
$\ulcorner SiP \urcorner$	,,	$\ulcorner P$ belongs to some $S\urcorner$
$\ulcorner SoP \urcorner$	,,	$\ulcorner P$ does not belong to some $S\urcorner$
$\ulcorner Np \urcorner$	,,	$\ulcorner$it is necessary that $p\urcorner$
$\ulcorner \Diamond p \urcorner$	,,	$\ulcorner$it is possible (cp. 10. 12) that $p\urcorner$
$\ulcorner Ep \urcorner$	,,	$\ulcorner$it is contingent (cp. 10. 11) that $p\urcorner$
$\ulcorner Ip \urcorner$	,,	$\ulcorner$it is impossible (cp. 10. 31 ff.) that $p\urcorner$
$\ulcorner Yp \urcorner$	means	that $\ulcorner p \urcorner$ is not qualified by any modal functor
$\ulcorner p\,(t) \urcorner$	for	$\ulcorner p$ at the time $t\urcorner$

The current number of a theorem is put in brackets when the corresponding formula is not stated but only indicated or used; an asterisk is added when it is stated by the ancient author with variables.

A list of the abbreviations of the titles of works quoted is contained in the bibliography; out of those abbreviations only the following are used in the text:

"*An. Post.*"	for	"Posterior Analytics"
"*An. Pr.*"	for	"Prior Analytics"
"*De Int.*"	for	"De interpretatione"
"*Principia*"	for	"Principia Mathematica"
"*Soph. El.*"	for	"De sophisticis elenchis"
"*Top*"	for	"Topica"

I. PROLEGOMENA

1. Introduction

This introduction is composed of two sections: in the first we shall explain the scope of (A) and the symbolism used in (B) the book; in the second an outline of the history of the History of Formal Logic will be given; we shall deal with that history in ancient and medieval times (C), during the XIXth century (D) and in recent years (E). A final paragraph will be devoted to the statement of the main problems awaiting study in our domain (F).

1 A. Aim and scope of the book

The present book is intended to supply mathematical logicians with a synthetic outline of the main aspects of ancient formal logic which are known in the present state of research. In order to avoid misunderstandings, each of the above terms has to be explained.

The reader is supposed to be a *mathematical* logician, i.e., to know both the symbolisms and the (English) language of contemporary mathematical logic; those who are not acquainted with it must be warned that several terms used in that language have a particular meaning, different from the meaning attributed to the terms of the same form in other contexts.

The subject of the book is *formal* Logic; by this we understand a science such as was developed by Aristotle in his Prior Analytics, i.e., essentially the theory of syllogisms as defined in *An. Pr. A* 1, 24*b* 18—20. Along with the syllogisms proper, the structure of the sentences and semiotics will be studied; contrariwise, not only all ontological, psychological and epistemological problems, but even methodological topics will be omitted in so far as possible. This is perhaps regrettable; but there are several good books on those subjects while there is *none* on ancient formal logic as a whole — and the limitation of space forced us to omit everything which was not strictly formal.

By *ancient* formal logic, Greek logic from the beginning of Greek Philosophy until the end of Antiquity is meant. We have, it is true, some Latin textbooks of formal logic — but they all seem based on, or even copied from, Greek sources. It is perhaps worthwhile mentioning that there is also an ancient *Indian* Logic; this lies, however, outside our present scope. [1]

What is offered here is an *outline*, moreover a very fragmentary one. A complete account of ancient formal logic cannot be written at the present date because of the lack of scientific monographs on individual logicians and topics. The initial aim of the author was to limit himself to a reassumption of monographs already published; in the course of the work he was compelled, however, to use some of his own unpublished researches on Aristotle and had the exceptional fortune of reading the manuscript of Dr Benson Mates' book on Stoic logic. He also collected some new data on other topics. In spite of this, considerable parts of ancient logic have hardly been touched upon — e.g. the logic of the Commentators — while others, Aristotle included, have been treated in a way which is far from being complete. On the whole, what the book contains may be considered as a kind of starting point for future research. Yet, it is hoped that even this will supply logicians with some information difficult to be found elsewhere and give a general idea of what the ancient logic was and how it developed.

1 B. Symbolism

All formal theses found in ancient logicians and referred to here are stated in a conventional symbolism; this is done for two reasons: because when stated in that way they are (a) much shorter, (*b*) more intuitive for mathematical logicians. The symbolism used is that of the *Principia* with a few changes and additions

[1] For bibliography of Indian Logic cp. C. Regamey, *Buddhistische Philosophie* (Bibl. Einführungen in das Studium der Philos., hrsg. v. I. M. Bocheński, 20/1), Bern 1950, NN. 25. 11—24. Among the works quoted there, however, only the papers by the late S. Schayer (N. 25. 22) are written by a mathematical logician.

which are listed at the beginning of the book. Here are some remarks which may be of importance for the correct understanding of our formulae.

Capital Latin letters are used *ambiguously* as names of classes or of properties. This method was followed because those are the symbols most used to stand for the capital Greek letters of Aristotle and his followers who did not distinguish between classes and properties. The traditional propositional functions $\ulcorner SaP \urcorner$ etc. are used because there is no adequate interpretation of the Aristotelian $\ulcorner P$ belongs to all $S\urcorner$ in the symbolism of the *Principia.* The existential quantifier "$(\exists x)$" is used also in an ambiguous way, meaning that an x exists, without specifying the kind of existence involved. The modal functors "N", "$\Diamond$", "E", "I" are meant to be *logical,* not metalogical functors; thus $\ulcorner Np \urcorner$ is to be read: $\ulcorner$it is necessary that $p\urcorner$, *not*: $\ulcorner\ulcorner p\urcorner$ is necessary$\urcorner$. Finally, the functor of implication "$\supset$" means here the every-day "if . . . then", not the material implication, except in the chapter on Stoic functors (15).

As far as the use of quotation marks is concerned, the general principle followed is that everywhere, where there was any doubt as to the status of the symbols, corners were used instead of usual quotes. In formulas such as $\ulcorner T \ulcorner p \urcorner \urcorner$ this is probably far from being correct; it is hoped, however, that this lack of rigour will not be felt as a handicap in the understanding.

The ancient *rules* (schemes of inference) have always been rendered in form of *laws* with the remark that the text contains a rule analogous to this law. By this the following is meant: let R be a rule; then L is said to be a law analogous to R if and only if L is a conditional in which the product of the premisses of R is asserted to imply the consequent of R (in R itself entailment if meant, of course). E.g. the first Stoic undemonstrated (16. 21) is a rule which may be stated, according to the modern use, in the form of

$$\begin{array}{c} p \supset q \\ p \\ \hline q. \end{array}$$

The corresponding law will be

$$p \supset q . \; p . \supset . q .$$

There is no risk of confusion here, as in all ancient rules the entailment occurs only once, and this is rendered by the *main* functor of implication in the corresponding law. It will be seen that this method both shortens the formulae and makes them more intuitive.

1 C. Ancient and medieval history of ancient logic

One meets sometimes with the assertion that history of philosophy is an invention of the XVIIIth century. This is in so far correct, that in older times — in spite of Aristotle's and Thomas Aquinas' explicit teaching — scholars neglected completely the genetic point of view in history of logic; on the other hand, there is no doubt that another aspect of historiography, namely the understanding of doctrines, was much cultivated by ancient and medieval thinkers. A complete account of ancient logic would have to take their results into consideration. Unfortunately, we know practically nothing of all the huge work which was accomplished, especially on Aristotle, by Greek, Syrian, Arabian, Jewish, or, above all, by Latin medieval logicians: as was already stated, the Greek commentators have not yet been studied, while the others are little more than a field for future research. And yet, we know that there *were* important discoveries during that time. This has been proved at least in one particularly striking instance: Albertus Magnus had a perfect understanding (superior to that of Alexander, not to mention Prantl) of the highly difficult Aristotelian modal logic. [2] This understanding has been nearly completely lost, however, during the modern ages. [3]

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[2] Bocheński, *Z historii* 29ff.; *Notes* 684ff. — [3] The Aristotelian modal logic and the Stoic-Megaric logic of propositions are striking instances. For the former see e.g. O. Hamelin, *Le système d' Aristote*, Paris 1920 and St. Dominczak, *Les jugements* (sic!) *modaux chez Aristote et les Scolastiques*, Louvain 1923 as compared with the medieval doctrines in Bocheński, *Z historii*; for the latter any "modern" treatise of logic, Prantl, Adamson as compared with the medieval teaching in Łukasiewicz, *Zur Gesch.*, Bocheński, *Consequentiae* and Dürr, *Aussagenlogik*.

1 D. State of the History of Formal Logic during the XIXth century

Modern history of Logic had been started during the XIXth century, but its state was very bad at that time — indeed until 1930 approximately — because of two phenomena. On one hand, most of the historians of logic took for granted what Kant said on it; namely that "formal logic was not able to advance a single step (since Aristotle) and is thus to all appearance a closed and complete body of doctrine" [4]; consequently, there was, according to them, no history of logic at all, or at the most, a history of the decay of Aristotelian doctrines. On the other hand, authors writing during that period were not formal logicians and by "logic" they mostly understood methodology, epistemology and ontology. That is why e.g. Robert Adamson could devote 16 pages to such a "logician" as Kant — but only five to the whole period from the death of Aristotle to Bacon, i.e. to Theophrastus, the Stoic-Megaric School and the Scholastics. In order to realize what this means, it will be enough to remember that from the point of view we assume here, Kant is not a logician at all, while the leading Megaricians and Stoics are among the greatest thinkers in Logic.

The worst mischief was done during that period by the work of Carl Prantl (1855). This is based on an extensive knowledge of sources and constitutes the only all-embracing History of Ancient Logic we have until now. Unfortunately, Prantl suffered most acutely from the two above-mentioned phenomena: he believed firmly in the verdict of Kant and had little understanding of formal logic. Moreover, he had the curious moralizing attitude in history of logic [5], and, as he disliked both the Stoics and the Scholastics, he joined to incredible misinterpretations of their doctrines, injurious words, treating them as complete fools and morally bad men precisely because of logical doctrines which we believe to be very interesting and original. It is now known that his work — excepting as a collection of texts (and even this far

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[4] *Kritik der reinen Vernunft.* 2d ed. p, VIII (English by N. Kemp Smith) —
[5] Cp. e.g. p. 488.

from being complete) — is valueless. But it exercised a great influence on practically all writers on our subject until J. Łukasiewicz and H. Scholz drew attention to the enormous number of errors it contains.

1 E. Recent research

We may place the beginning of recent research in our domain in 1896 when Peirce made the discovery that the Megaricians had the truth-value definition of implication. The first important studies belonging to the new period are those of G. Vailati on a theorem of Plato and Euclid (1904), A. Rüstow on the Liar (1908) and J. Łukasiewicz (1927); the Polish logician proposed in it his re-discovery of the logical structure of the Aristotelian syllogism and of Stoic arguments. Four years later appeared the highly suggestive, indeed revolutionary, History of Logic by H. Scholz, followed in 1935 by the paper of Łukasiewicz on history of logic of propositions; this is considered until now as the most important recent contribution to our subject. Both scholars — Łukasiewicz and Scholz — formed small schools. J. Salamucha, the pupil of the former, wrote on Aristotle's theory of deduction (1930) and the present author on the logic of Theophrastus (1939). Fr. J. W. Stakelum, who studied with the latter, wrote a book on Galen and the logic of propositions. On the other hand, A. Becker, a student of H. Scholz, published an important book on Aristotle's contingent syllogisms (1934). Professor K. Dürr was also influenced by Łukasiewicz in his study on Boethius (1938); his results were somewhat improved by R. van den Driessche (1950). In the English speaking world we may mention the paper of Miss Martha Hurst (1935) on implication during the IVth century (1935) — but above all the already quoted work of Dr B. Mates on Stoic Logic (in the press), which, being inspired by Łukasiewicz and his school may be considered as one of the best achievements of recent research.

Such is, in outline, the work done by logicians. On the other hand philologists had considerable merits in the study of ancient logic. We cannot quote here all their contributions, but at least the important book of Fr. Solmsen (1929) on the evolution of Aristotle's logic and rhetoric must be mentioned, and, above all, the masterly commentary on the Analytics by Sir W. D. Ross (1949). It does not always give full satisfaction to a logician trained on modern methods, but it is, nevertheless, a scholarly work of a philologist who made a considerable effort to grasp the results of logicians.

1 F. Tasks for the future

In spite of these studies nearly everything is still to be done in history of ancient formal logic. Not even the texts are sufficiently studied. The most urgent needs as far as they are concerned is a critical edition of the Stoic-Megaric fragments, the *Stoicorum Veterum Fragmenta* of von Arnim being now insufficient. Even Aristotle's text is not satisfactory: we still need a new edition of *De Int.* and of the Topics, while, in spite of the excellent work done by Sir W. D. Ross, more studies seem to be required on the Analytics.

But above all, monographies on the logical doctrines are needed. Here is a list of subjects which have hardly been touched by an expert hand: pre-socratic dialectics; Plato's formal logic; the logic of Topics; the assertoric syllogistic of Aristotle; his semiotics; the syllogism based on hypothesis; the peripatetic school after Theophrastus; Sextus Empiricus; Galen (Fr. Stakelum studied only the *Institutio Logica*); Alexander of Aphrodisias; Porphyrius; Ammonius; Boethius (a thorough examination of all his logical works); Simplicius; Philoponus; Apuleius; Cicero. These are only the *main* topics which await a scientific inquiry — along with them a number of less important ones should be studied.

As far as problems are concerned we are still far from understanding the true nature of pre-Aristotelian and early Aristotelian logic, the rise of the syllogism, the origin of the Stoic-Megaric doctrines, or the development during the centuries which followed Chrysippus. Also the connection and mutual influences of mathematical and logical studies are hardly known.

And once this work has been done, everything which has been elaborated until now — above all the content of the present outline — will probably have to be re-examined and restated. Ancient formal logic is actually little more than a field for scientific research.

2. General Survey

2 A. The succession of thinkers and schools

The history of ancient philosophy covers about eleven centuries, from Thales who lived during the sixth century B.C. to Boethius and Simplicius who flourished at the beginning of the sixth A.D. From the point of view of the history of formal logic this long epoch may be divided into three periods.

(1) The pre-Aristotelian period, from the beginnings to the time at which Aristotle started writing his Topics (about 340 B.C.). There is no formal logic during this period, i.e. no *study* of logical rules or laws; but some of them are *used* consciously since Zeno of Elea, and Plato tries, if unsuccessfully, to build up a logic.

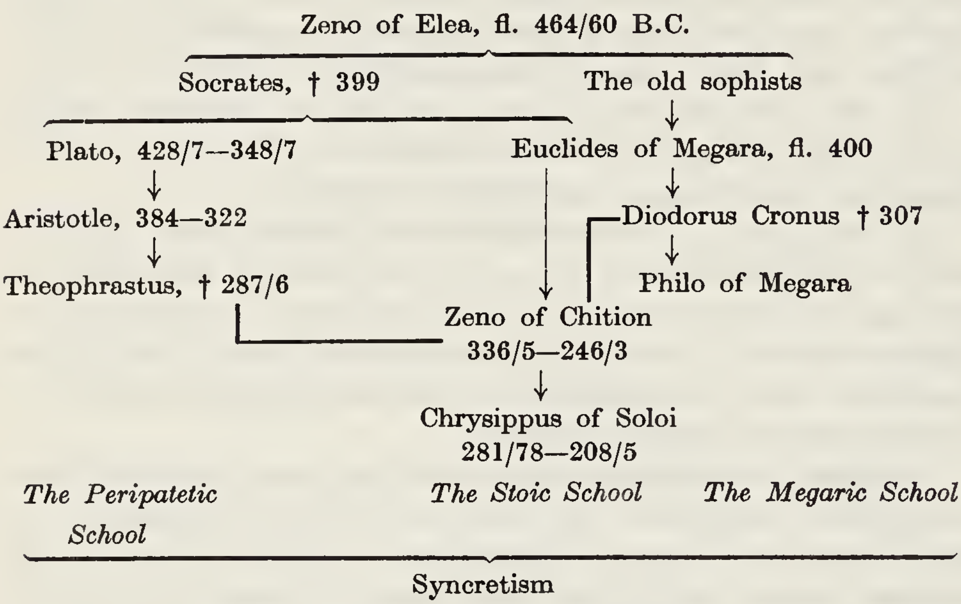
(2) The creative period, from the time of Aristotle's Topics to the death of Chrysippus of Soloi (205/8 B.C.). During this period Logic was founded and considerably developed.

(3) The period of schoolmasters and commentators, from the death of Chrysippus until the end of Antiquity. In that period no more creative work is done, as far as we know; moreover, a continuous decline of formal logic seems to take place. Boethius and Simplicius who are considered as the last ancient philosophers are also the last ancient logicians.

It appears, consequently, that out of the eleven centuries mentioned above only about 150 years are of real importance; but those years are of enormous importance — they are, indeed, among the best years of logic in the whole history of humanity until now.

The succession of different trends of logical thought — for there were several such trends — can be briefly stated in the following terms. If Zeno is, according to Aristotle, "the inventor of dialectics", Socrates seems to have been the real father of formal logic; at least both Plato and Euclides, the head of the Megaric School, claim to be his disciples. Plato was the teacher of Aristotle, the founder of formal Logic; Aristotle was succeeded by Theophrastus, Eudemus and some others, who, if far less important than he, are nevertheless productive logicians. This is one line of development of logic, the peripatetic. The other line starts with Euclid of Megara and in the second generation after him bifurcates into the properly Megaric School, with Diodorus Cronus, and Philo of Megara his pupil, as most important logicians on one hand — the Stoic School founded by Zeno of Chition and having as chief thinker Chrysippus of Soloi on the other. After Chrysippus' death one hears no more of the Megaricians, and, later on, a syncretism of the Peripatetic and Stoic-Megaric Schools appears.

Here is a scheme which may help in comparing the respective dates and mutual influences; it contains only the most important names:



2 B. The main lines of evolution

Such is the external history of Greek logic. As far as its content is concerned, the evolution seems to have been the following.

(a) First of all, logical rules were used without being explicitly formulated or even known as such; then, and such seems to have been the case of Zeno of Elea, those rules become consciously applied, but still they are not formulated or studied. A third stage is represented by the explicit formulation of rules without any special apparatus: this we find in the Aristotelian Topics.

Further on, technical means useful for the study of logic are introduced by Aristotle in his Prior Analytics, namely variables and a peculiar terminology; at this stage laws are not yet distinct from rules. The fifth and last stage is represented by a clear distinction of both, such as we find in fragments of the Stoics.

(b) As far as the formulae themselves are concerned, we may also distinguish several stages of evolution.

(α) The pre-analytic type. Those are formulae which we would consider as rules corresponding to laws of the logic of (unanalysed) propositions, such as the *modus ponendo ponens*. There is here no analysis of the sentence, no distinction between universal and particular sentences, no knowledge of the exact role of the subject and predicate and so on. But — and this is very important for the understanding of that stage — the logicians who use or even study such formulae are not thinking about them in such an abstract way as the Stoics did when they stated their undemonstrated. The proof that it was so, is found in the fact that the atomic sentences involved in their formulae all have subjects of the same form. Thus, those logicians will not think of the law

$$\text{(A)} \quad p \supset q . p . \supset . q$$

but rather of the more special case, being a substitution of (A), namely:

$$\text{(B)} \quad Ax \supset Bx . Ax . \supset . Bx$$

Such is the situation in all pre-Aristotelian writings we know, and to a large extent also in Aristotle's Topics.

(β) The analytic type. To this type belong the formulae elaborated by Aristotle in his Prior Analytics. Here, an exact analysis of the structure of atomic sentences is effected; their quantification is considered, and the relative positions of subject and predicate examined. There seem to have been several stages of evolution in that period; from that — rather primitive — represented by some texts of the Topics, through the analysis of the sentence into a subject, a predicate, the copula and the quantifier, until the highly refined analysis, somewhat similar to that which we find in the recent formal implication (in some chapters of the modal logic and also in *An. Pr. A*41). We have here to do with a relatively highly developed logic of classes. The validity of the laws studied is based on the internal structure of the atomic sentences involved. On the other hand there is still very little of a logic of sentences.

(γ) Finally we find the most abstract formulae, such as are represented by the Chrysippian undemonstrated. Corresponding laws are to be found already in the Prior Analytics — there are even some with propositional variables. But there is no theory of such laws and no effort to define, say, the implication. In our third period, on the contrary, which is the Stoic-Megaric, abstractly formulated laws and rules of the logic of propositions are studied for themselves and the exact meaning of the sentence-determining functors, such as the functor of implication or of alternative, are discussed.

(δ) But this third period does not seem to have had a long duration. As far as we know, very soon after the death of Chrysippus more and more confusion invaded the field of logic. All has not been lost — especially, the doctrine of the first seven chapters of *An. Pr. A* was universally known, and with it some fragments of Stoic logic; but it seems as if later logicians were more and more driving toward the initial, pre-analytic type of logic. In spite of some brilliant exceptions in both the Aristotelian and Stoic-Megaric camp, the last period of Greek logic seems to have been, on the whole, one during which very few people understood the meaning of what has been done during the second and the third.

2 C. The struggle of schools

There is in the latter part of the history of Greek Logic, perhaps already since the time of Theophrastus, a curious, and from the point of view of the interest of logic, a very unfortunate phenomenon: the bifurcation of logical research into two schools, neither of which seem to have understood that there was no real opposition between their *logical* tenets, that they were both working in different departments of the same science. I mean the struggle between the peripatetic and the stoic-megaric line of thought. From the point of view of scholastic logic, such as developed during the XIIIth century, and again from the point of view reached recently, there is no opposition between logic of propositions and logic of predicates: both are legitimate parts of formal logic and no complete logic can avoid stating laws or rules belonging to both. Aristotle, as it seems, was plainly aware of that fact, as he explicitly recognised the legitimacy of rules corresponding e.g. to the law

$$pq \supset r. \supset .p \sim r \supset \sim q.$$

He only thought — quite rightly from his methodological point of view — that such laws or rules cannot be used in demonstration as he defined it. But later logicians did not understand the situation as he did. They thought — this is at least what we know from Alexander and Galenus — that Stoic and Aristotelian formulae belong to two mutually opposed logics.

Consequently there was a continuous struggle between both schools. The Peripatetics tried to "reduce" the Stoic rules to Aristotelian laws and Stoics seem to have completely neglected the logic of predicates and classes. And as, in the long run, the very abstract Stoic Logic — which moreover, being deprived of a logic of predicates, was of little use — lost the battle, we find at the end of Antiquity a curious regress, not to Aristotle at his best, but sometimes even to the pre-analytic type of logic.

This regress might be considered as the fourth period of evolution of Greek logic. It is true that, as we said, some elements of both Aristotelian and Stoic teaching were preserved — but the blending of both which aimed at a "victory" of the logic of predicates over that of propositions caused logic, on the whole, to decay.

II. THE FORERUNNERS

3. Formal Logic before Aristotle

Aristotle is the first ancient formal logician; the earlier thinkers may all be considered as forerunners of that science. They developed, however, some rudimental semiotic doctrines (A) and used — it seems consciously — several logical rules (B). Plato appears to have conceived what formal logic should be and tried to build it up, but without success (C). This period is still little known, as few studies have been made on it by logicians; [1] therefore we are obliged to limit ourselves to very generic remarks.

3 A. Semiotics

Syntax and, later on, Semantics seem to have been more cultivated in pre-Aristotelian times than logic proper. Thus Plato reports [2] that Prodikos of Keos (born 460/70?) had been concerned with "the right use of words". Protagoras of Abdera (fl. 444/3 B.C.) is said to have classified the expressions (*λόγους*); according to Diogenes [3] he divided sentences into prayers, questions, answers and commands. Aristotle says [4] that he also distinguished the genera of the noun. The fullest account of that syntax is to be found in Plato's *Sophist* — probably not an original theory of Plato himself. The very problem of syntax is stated there: "whether all names can be connected with one another, or none, or only some of them". [5] Names are divided into verbs "which denote actions" and nouns i.e. "marks set on those who do the actions".[6] A sentence is a string of words in which "verbs are mingled with nouns" [7]; every sentence must have a subject [8]; a combination of a noun with a verb is "the first and smallest" phrase. [9]

Semantics offered great difficulties to those thinkers, as the

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[1] Cp. However, Krokiewicz, Scholz, *Klassische Philosophie,* Vailati; also Solmsen, *Entstehung, Discovery* and Robinson are of importance for Plato. — [2] *Crat. 384 b, Euth. 277 e ff.* — [3] *DL 9, 53f.* — [4] *Rhet. Γ 5, 1407 b 6.* — [5] *Soph. 261 d.* — [6] ib. *261 d—262 a.* — [7] ib. *262 c.* — [8] ib. *262 e.* — [9] ib. *262 c.* —

theory of Antisthenes shows.[10] Aristotle reports that according to him "nothing could be described except by the account proper to it (οἰκείῳ λόγῳ), one predicate to one subject".[11] The 'what' cannot be defined, as the definition is just a "long rigmarole".[12] Thus it seems that every sentence was a tautology for Antisthenes. Another instance of such perplexity is supplied by the doctrine of Lycophron, who is said by Aristotle[13] to have avoided the use of the copula "ἐστί", fearing that the One be confounded with the Many. A better, but still rather elementary, semantics is to be found in Plato, probably again not his original work. In the *Cratylos* he discusses the nature of speech and reached a basically conventionalistic theory. Thought, he says, is "akin to language", indeed, "a conversation of the soul with itself"[14] while the speech is "the stream of thought which flows through the lips and is audible".[15] Every sentence has a quality, namely falsehood and truth.[16] A false sentence is one "which asserts the non-existence of things which are and the existence of things which are not".[17] The full semantic theory of Plato is so strictly connected with his metaphysical and epistemological views, that we cannot deal with it here. It is, however, not improbable that Plato had already the main ideas of Aristotle's semantics.

3 B. Pre-Aristotelian use of logical rules

Aristotle says that "on the subject of reasoning" he "had nothing else on an earlier date to speak about"[18]; in fact we know of no *correct* logical principle *stated* and *examined* for its own sake before Aristotle. Some logical rules were, however, *consciously used*, at least since Zeno of Elea (fl. 464/60 B.C.) whom Aristotle is reported[19] to have called "the inventor of dialectics". When examining such rules in the preserved fragments, we are induced to advance several conjectures of a certain importance for the

[10] There is no evidence that Aristotle alludes to him *Met. Γ 1005 b 2—5*; *4, 1006 a 5—8*; *5, 1009 a 20—22*; *6, 1011 a 12—20*; but he is explicitly quoted *Met. Δ* and *H.* — [11] *Met. Δ 28, 1024 b 32—34.* — [12] *Met. H 3, 1043 b 23ff.* — [13] *Phys. A 2, 185 b 25—32.* — [14] *Soph. 264 a.* — [15] *ib. 263 e.* — [16] *ib. 262 eff.* — [17] *ib. 240 e.* — [18] *Soph. El. 34, 184 a 9ff.* — [19] *AM 7, 7*; *DL 8, 57*; *9, 25.*

understanding of the origin of Greek formal logic. (1) That logic seems to have risen out of *dialectics*; now dialectics means at that time a discussion, a dialogue in which the opponent tries to refute some assertion. Consequently, (2) the principles we find in use of those old authors are in majority different forms of the principle of *reductio ad absurdum*, apagogic rules. (3) Most of them were certainly conceived rather as *rules* than as logical laws — but it must be stressed that at that time nobody would have thought of distinguishing both. (4) Finally the principles used seem also to have been thought rules of *logic of terms*, not of propositions. Here are some such rules extracted from different sources:

The words of Zeno reported by Simplicius (a good authority) show that the Dialectician used a rule corresponding to the law:

[3. 1.] $$Ax.\supset.Bx.Cx : \sim(Bx.Cx) :\supset: \sim Ax$$ [20]

In Plato we find an analogon of

[3. 2.] $$Ax \supset \sim Ax.\supset.\sim Ax$$ [21]

and the same seems to be ascribed to Democritus by Sextus [22] with a technical (Stoic) interpretation. The early Aristotelian dialogue *Protrepticus* contained a reasoning according to

[3. 3.] $$Ax \supset Ax.\sim Ax \supset Ax.\supset.Ax$$ [23]

or perhaps a simpler form of the same, also used later on by Euclid [24]:

[3. 4.] $$\sim Ax \supset Ax.\supset.Ax.$$

[20] *Simpl. Phys. 140, 34*; *D 1, 255.* — [21] *Theaet. 171 a.* This was discovered in 1904 by Vailati. — [22] *AM 7, 389*; *D. 114, 111, 15.* Discovered by Scholz; Professor E. W. Beth drew the attention of the author to this point and supplied the reference. — [23] *Rose Fragm. 51. p. 56ff.*; discovered by Scholz. The tradition is rather confused. Alexander (*Rose 56 26ff.*) and Lactantius (*58, 11ff.*) have $\ulcorner\sim Ax \supset Ax.\supset.Ax\urcorner$; an anonymous scholiast (*57, 10ff.*) $\ulcorner Ax \supset Ax.\sim Ax \supset Ax.\ Ax \vee \sim Ax.\supset.Ax\urcorner$; Olympiodorus (*57,13ff.*), David (*57, 29*) and Elias (*57, 20*) $\ulcorner Ax \supset Ax.\sim Ax \supset Ax.\supset.Ax\urcorner$ (the former two in an elliptic form.) — [24] *Prop. IX, 12*; discovered by Vailati.

Zeno used also consciously, as it seems, a rule corresponding to

$$[3.5.] \qquad Ax \supset Bx . Bx \supset Cx . \supset . Ax \supset Cx. \text{ [25]}$$

More such rules could be extracted from the great fragment of Gorgias; [26] as, however, this text not only contains typically Stoic technical terms, [27] but also betrays a very high level of logical skill unthinkable at that period, the principles used in it cannot safely be ascribed to the Sophist. Perhaps something like 3. 1, a kind of destructive dilemma, and the principle of contradiction:

$$[3.6.] \qquad (x) . \sim (Ax . \sim Ax) \text{ [28]}$$

might have been used by him. The above list is by no means exhaustive; further study would probably discover more such material, but it seems that it would all belong to the same class of rules as those quoted by us.

3 C. Plato's dialectics

Plato's position in the history of logic is a rather complex one. His dialectics appears to us as being a confusion of different sciences and different methods. It includes on one hand the art of disputation, metaphysics and logic; on the other hand Plato does not distinguish between formal logic, methodology (of a kind rather akin to that of empirical sciences of to-day) and the intuitive approach to (mostly) axiologic problems. The reading of his dialogues is almost intolerable to a logician, so many elementary blunders are contained in them. [29] It will be enough to mention his struggling with the false principle $\ulcorner SaP \supset \bar{S}a\bar{P} \urcorner$ [30] or the difficulty he has in grasping that who does not admit $\ulcorner SaP \urcorner$ must not necessarily admit $\ulcorner SeP \urcorner$. [31]

And yet Plato's work has an enormous importance in the history

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[25] *Simp. Phys. 140, 27ff.*; *D 3, 257f.* — [26] *AM 7, 66ff.*; *D 1, 280*; cp. Untersteiner (*123f.*) and Dupréel (*53*) neither of whom cares, however, for logical problems. — [27] Cp. *Diels p. 281, 28*! — [28] *AM 7, 67.* — [29] Cp. Robinson. — [30] e.g. *Gorg. 507 a*; *Alc. I, 126 c*; Cp. Robinson. — [31] e.g. *Gorg. 466 a*; *Meno 73 e*; *Prot. 350f.*

of logic for several reasons. (1) He was the first to conceive and to state clearly the ideal of valid laws of reasoning. [32] (2) Following Socrates, to whom Aristotle says "inductive arguments and universal definition" "may be fairly ascribed" [33], he seems to have shifted attention from the apagogic proofs to positive demonstrations of statements attributing a propriety to a subject, thus opening the line of "peripatetic" logic, i.e. logic of terms. (3) One of his methods, namely that of division (*διαίρεσις*) became the origin of the syllogism. It consists in dividing a genus into two species, finding to which of them the subject belongs, dividing this again and so on. It is not only explicitly recommended, but even experimentally tried before it was applied by Plato. [34] As Aristotle has shown [35] the division is a "weak syllogism" (*ἀσθενὴς συλλογισμός*): as a matter of fact, it does not prove anything, but consists in a series of assumptions. (4) Finally nearly everything in Aristotle's logic, if we except the analytical syllogism and some doctrines connected with it, is most probably a reflex elaboration and development of procedures used already, at least in a rudimentary way, by Plato. [36] Correct logic we find none in his work; he was, however, a thinker who during his whole life was searching for logic and paved the way for its founder.

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[32] *Tim. 47 b.* — [33] *Met. M 4, 1078 b 27—29.* — [34] *Soph. 218ff.* — [35] *An. Pr. A 31, 46 a 31ff.* — [36] This is most evident for the "hypothetical" syllogism; cp. *Meno 86 e—87 c, Prot. 355f.*

III. ARISTOTLE

There is no thinker in the whole history of formal logic whose importance, both historical and systematic, can be compared with that of Aristotle. For not only is he the Logician who was first to *state* formal laws and rules and study them for their own sake, but also he did it in a way which, given that he is the originator of the whole subject, appears as a tremendous achievement. It has been said that not a single psychological general category was invented after Aristotle; and the same is perhaps true of formal Logic. Of course he did not invent the whole of it; but we owe him most of the fundamental ideas on which Logic is still working today — such as the idea of a formal entailment, of a variable, of an axiom, and many others. At the same time Aristotle's Logic dominated western Philosophy during more than twenty centuries — another fact unique in its importance. Moreover, he is the only great ancient logician whose works are preserved.

It will be consequently only reasonable to devote to Aristotle's teaching more space than to any other ancient logician. We shall deal with that teaching in 8 chapters: (1) Life, work and evolution, (2) Notion of Logic, (3) Topics, (4) Theory of opposition; principles of contradiction and of excluded middle, (5—6) Assertoric syllogism, (7) Modal syllogism, (8) Other doctrines, including the hypothetical syllogism.

4. Life, work and evolution

Logic accomplished more progress during Aristotle's life than in any other period of Antiquity. We shall give therefore some details concerning his life (*A*), his extant logical works (B), the criteria of their chronology (C), the chronology itself (D), and finally we shall sketch the main lines of his evolution (E).

4 A. Life [1]

Aristotle was born in 384/83 B.C. in Stageira; from 367/66 to 348/47 he was a member of Plato's Academy. After Plato's death he went on journeys (348/7—343/42; Assos and Mytilene). From 343/4 to 336/35 he was the teacher of Alexander the Great. Then, after a year's stay in Stageira, he came back to Athens (335/34) and remained there as head of his "peripatetic" school until the death of Alexander (323). At that time he returned to Chalcis and died one year later in 322/21.

According to those external circumstances Aristotle's life may be divided into three main periods: (1) Academic 361/66—348/47, (2) of travels 348/47—335/34, (3) second Athenian or Master-period 335/34—323.

4 B. Logical Works

The surviving logical works of Aristotle [2] were arranged by Andronicus Rhodos (1st century B.C.) into six books; this body was later on called "Organon" (i.e. "instrument") by the Byzantine

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[1] Cp. Jaeger; Ross, *Aristotle*; bibliography in Philippe, Nos 2. 11ff. — [2] The only comprehensive modern work on Aristotle's logic, which still retains considerable value, is that of Maier. Among the commentaries, those of Alexander of Aphrodisias and of Sir W. D. Ross on Analytics are the most important among the known; there is, however, no doubt that several older commentaries should be consulted, e.g. those of Albert the Great and of Zabarella. For bibliography cp. Philippe. Works bearing on particular problems are quoted in the respective chapters. The best modern English translation is that under the direction of Sir W. D. Ross; H. Scholz (*Geschichte* p. 27) says that this is the *only* modern translation which may be recommended; and yet it does not satisfy a logician on all points.

logicians. Besides the Organon there is one more completely logical work, namely the fourth book (*Γ*) of Metaphysics. All those works seem to be drafts of lectures, and not destined for publication in the present form. All with the exception of the Categories (which are probably spurious, [3] but nevertheless seem to contain Aristotelian doctrines) are authentic. Here is a list of them with a short description:

1. *Categories* [4] (In Bekker's edition pp. 1—15); deals with (1) homonyms, etc., (2) (ch. 1), predication and categories (3—9), different notions (the so-called "postpraedicaments", ch. 10—15).

2. *On Interpretation* [4] (pp. 16—24). Contains Aristotle's syntax (ch. 1—5) and his early theory of negation and opposition (6—14).

3. *Prior Analytics* [5] (pp. 24—70). Two books: *A* and *B*. Contains the theory of the categorical syllogism (*A1, 2, 4—7*), of the modal syllogism (*A3, 8—22*) and considerations on the system of syllogism with remarks on other laws and methods (*A 24—46, B*). This is probably the most original work ever written on logic; and, as Prof. Scholz says, it remains still the best possible introduction to its study.

4. *Posterior Analytics* [5] (pp. 71—100). Two books: *A* and *B*. Discusses demonstration, definition, deductive method and some psychological problems; it is rather a treatise of methodology than of formal logic. In spite of some remarkable doctrines it contains, this work is evidentally a collection of separate notes, some of which are very confused.

5. *Topics* [6] (pp. 100—164). Eight books: *A*, *B*, *Γ*, *Δ*, *E*, *Z*, *H* and *Θ*. A treatise of "dialectical" reasoning in which "commonplaces" (*τόποι*) useful for discussion are stated and the method of it studied. A relatively well-written book, perhaps the only one in the Organon which was nearly finished by the author.

6. *On Sophistical refutations* [6] (pp. 164—184). Is now considered as the ninth book (*I*) of Topics, as it contains at the end a survey of the whole work. Discusses fallacies and their solution.

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[3] Cp. Dupréel, *Aristote*. — [4] The best text available is that of Minio Paluello. —[5] Best text by Ross. — [6] Best text by Strache-Wallies.

7. Fourth book of *Metaphysics*[7] (*Γ*) (pp. 1003—1012). Discusses the principle of contradiction in a violent polemic style.

4 C. Criteria of chronology

There is no doubt that Aristotle's logic did evolve considerably; it is also certain that none of his works remained unchanged. Furthermore, it seems that most of them were placed at a later date by Aristotle himself into a general frame, corresponding somewhat to that retained by tradition. Thus the disentangling of the different steps in his evolution is a difficult task. Several criteria can be used however. Two of them are quite certain: (1) the use of the analytic (categorical) syllogism, which is a late invention, (2) the use of letters as abbreviations and as variables.[8] Along with these, the following less certain criteria may be applied: (3) the level of logical rigor and style, which is very different in different writings and might be supposed to improve with time, (4) the refinement of the analysis of the sentence, from the simple "S—P" scheme, through the "all (none, some) of the S is P", to the highly complex "that to all of which applies S, to that all applies also P"[9]; (5) the letters probably occur first as simple abbreviations, then as term-variables, and only at the end as propositional variables; (6) the modal sentences, which correspond to Aristotle's own philosophy of becoming, seem to be a later invention. On the other hand the criterion of diminishing Platonism and increasing formalism does not seem to be sufficiently substantiated.[10]

4 D. Chronology of works

By applying these criteria we find that the logical works of Aristotle — at least as far as the bulk is concerned — may have been

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[7] Best text by Ross. — [8] Those two criteria were particularly emphasized by Solmsen, *Entwicklung*. — [9] *An. Pr. A 41, 49 b 14ff.*; *13, 32 b 25ff.*; *An. Post. A 21,82 b 5ff.* — [10] That criterium, emphasized by Jaeger, was applied to the logical works of Aristotle by Solmsen, *Entwicklung*; the general theory of Jaeger is now rejected, however, by most scholars; cp. the bibliography in Philippe, Nos. *7. 11ff.* — about logical works also Ross, *Analytics*, Introd. ch. 2, p. *6ff*, and ch. 6 p. *75ff*.

written in the following order. (1) The Topics (with the *Soph. El.*) come first.[11] They contain no letters at all, the syllogism is not yet known, the level is low and the analysis of a sentence is rudimentary. (2) *Met. Γ* seems to belong to the same period; Aristotle is preoccupied here with the problem of contradiction and deals with it without letters, committing logical errors which he simply could not have committed later on. (3) *De Int.* must have been written later on; it contains semiotic doctrines which seem essentially Platonic; there are still neither letters nor syllogisms, but the level is noticeably higher than that of the *Top.* There is a chapter on modalities — yet this is still primitive in comparison with *An. Pr.* (4) In spite of some doubts, the present author thinks that *An. Post. B* may be placed immediately thereafter.[12] Aristotle knows here already both the syllogism and the letters; but these are *always* used as abbreviations only; the level seems inferior to that of other parts of the Analytics. (5) Thereafter, *An. Pr. A 1, 2, 4—7, 23—46* and perhaps *An. Post A* should be placed. We have here not only letters, mostly used as class-variables, and the syllogism is fully explained, but also the technical level of the thought and speech is remarkable. (6) The last logical works of Aristotle are perhaps *An. Pr. A 3, 8—22* and *B*. We find here the most refined doctrines, such as that of modality, and letters are sometimes used as propositional variables.

In any case, two periods can be distinguished with certainty: (1) *Top, Soph. El., Met Γ, De Int.* (2) *Analytics.*

4 E. Survey of the evolution

In the light of the above chronology the evolution of Aristotle's formal logic may be stated in the following way: (1) He first elaborated the Platonic λόγους (*Top., Met. Γ, De Int.*), considerably

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[11] So already Maier and especially Solmsen, *Entwicklung.* — [12] The order proposed by Solmsen is: *Top A—H, An. Post. A, Top Θ, Soph. El., An. Post. B., An. Pr.* But Ross, *Analytics,* seems to have definitively established the chronological priority of *An. Pr.* in regard to *An. Post. A*; the present author does not think that he equally succeeded in showing that *An. Post. B.* were written after *An. Pr.*

developing and explicitly stating the rules or laws on which they are based. By doing so he stated a wealth of interesting logical principles of which, however, *none* is an analytic syllogism. This period may coincide with that of travels (348/7—335/4). — (2) Later on he made his two great discoveries: that of the analytical syllogism and that of the variable. He then declared that the other (non-analytic) laws and rules are of lesser importance and concentrated on syllogism, first assertoric, then modal. — (3) By analyzing the axiomatic system of the former (he did not have time, as it seems, to do so with the latter) he discovered several metalogical rules and even some laws of the logic of propositions. These last discoveries were, however, not systematized by him.

5. Notion of Logic; semiotics

This chapter contains a short description of doctrines which would probably be considered by Aristotle himself as introductory to logic, namely of his theory of logic (A) and methodological doctrines (B), his syntax (C), semantics (D), and his teaching on truth (E). Most of those points are so strictly connected with the whole of Aristotle's philosophy that it is impossible to enter into details without raising a great number of problems alien to formal logic; at the same time, most of what is said by Aristotle on those topics is curiously incomplete, so much so, that if one considers only his explicit teaching, he might get a wrong idea of what Aristotle thought.

5 A. Logic

Aristotle's term for "logical" is "*ἐκ τῶν κειμένων*" [1], i.e. "following from the premisses", or "*ἀναλυτικός*" [2], while the term "logical" ("*λογικός*) in his works generally means the same as "dialectical", i.e. "probable". [3] Logic seems to have no place in Aristotle's system of sciences [4], and was perhaps considered by him rather as an "instrument" which must be learned before those sciences. [5] This does not mean, however, that Aristotle would not consider logic as a theoretical discipline — the very fact that he devoted so much effort and space to apparently useless logical problems shows that this was not the case.

We find no definition of logic in the preserved Aristotelian works; but its subject is clearly the syllogism and this is twice defined as a "*λόγος* in which some things being laid down, something different from them necessarily follows because of those laid down things" [6] (or: "because they are such" [7]). This is in fact a definition of deduction in its whole generality. The remarkable thing about

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[1] *An. Post. A 32, 88 a 18* and *30.* — [2] *An. Post. A 22, 84 a 7f.* and *b 2.* Cp. Scholz, *Geschichte 6f.* — [3] *An. Pr. B 16, 65 a 36*; *A 30, 46 a 9*; *B 23, 68 b 10*; *Top. A 1, 100 a 22, 29*; *Θ 11, 161 a 36* cp. Bonitz *183.* — [4] *Top. Z 6, 145 a 15f.*; *Met. E 1, 1025 b 25*; *Met. K 7, 1064 b 1.* The *λογικαί* in *Top. A 14, 105 b 19ff.* means clearly "epistemological". — [5] *Met. Γ 3, 1005 b 2—5.* — [6] *Top. A 1, 100 a 25.* — [7] *An. Pr. A 1, 24 b 18f.*

it is that it does not attribute to the syllogism any definite status: for "*λόγος*" may mean equally well a verbal discourse, a train of thought, or an objective structure (of the kind of the Stoic *λεκτόν*), while exactly the same is true of the *προτάσεις* and *ὅροι* of which the syllogism is said to be composed. There are, however, two texts which might perhaps throw some light on that point. In the *An. Post.* [8] Aristotle says that the demonstration is not about words but about things in the soul; and, while the whole structure of the *De Int.* and the *Top.* supposes that logical formulae are sequences of spoken words [9], it is asserted in the former that the laws hold about the "spoken affirmations" because similar laws hold with regard to the "judgements of the mind". [10] Thus we may say that for Aristotle logic is primarily an affair of right thinking and, secondarily, a matter of correct speaking.

5 B. Methodological doctrines

It will be necessary to state here some of the methodological doctrines of Aristotle which have a bearing on his formal logic.

(a) Since all knowledge starts, according to Aristotle's general philosophy, with particulars, induction has a fundamental importance in his methodology. As a matter of fact, we find in the Organon a theory of induction; the Greek word for it is *ἐπαγωγή*. Aristotle holds that every belief comes through syllogism (*συλλογισμός* = deduction) or through *ἐπαγωγή* [11]; the latter is defined as "a passage from particulars (*καθ' ἕκαστα*) to the universal (*τὸ καθόλου*)" [12]. It must be stressed, however, that the word has at least three different meanings, namely (1) didactic induction, (2) abstraction of universal concepts from particular sensations [13], (3) induction proper, which is again either complete [14] or generalizing induction [15]. While Plato seems to have considered the last variety as a logically valid procedure, Aristotle emphatically denied its validity [16] — which does not mean, of course, that he rejected the use

[8] *An. Post. A 10, 76 b 24f.* — [9] Cp. e.g. *De Int. 4, 16 b 26ff.* — [10] *De Int. 14, 23 a 32ff.* — [11] *An. Pr. B 23, 68 b 13f.* — [12] *Top. A 12, 105 a 13.* — [13] *An. Post. B 19, 100 a 3—100 b 5.* — [14] *An. Pr. B 23, 68 b 27ff.; 24, 69 a 17ff.* — [15] *Top. A 12, 105 a 14ff.* — [16] *An. Pr. B 23, 68 b 15—29.*

of the generalizing induction; the whole of his work is full of its applications to different problems.

In the *Top.* the syllogism itself is divided into demonstrative (*ἀποδεικτικός*) dialectical (*διαλεκτικός*) and sophistic (*ἐριστικός*) [17]; the difference between them consists in the quality of the premisses which must be necessary, true and prior to the conclusion in demonstration [18], but are probable only in the dialectical syllogism [19] and false in the sophistical. The latter may also be formally wrong. [20] Except for the last point, this division is irrelevant for formal logic as Aristotle explicitly says; [21] but it is important to notice that for Aristotle the scope of formal logic was to supply means for demonstration [22]. Now according to him, a demonstration proves through the definition of the subject that a property belongs to it. This is important, because it explains Aristotle's insistence on the "categorical" syllogism — since, from the point of view of his theory of demonstration only such a syllogism (and indeed only one in the first figure) was a useful logical tool.

5 C. Syntax

De Int. 1—5 and *Poet. 20* contain the following classifications of symbols:

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[17] *Top. A 1, 100 a 27ff.* — [18] but cp. *An. Post. A 2, 71 b 20ff.* where more conditions are added. — [19] *Top. A 1, 100 a 29f.*; cp. *14, 105 b 31.* — [20] *Top. A 1, 100 b 23ff.* — [21] *An. Pr. A 1, 25–28.* — [22] Cp. *An. Pr. A 1, 24 a 10f.*; and *4, 25 b 26ff.*

- Significant sound-symbol [23]
  - atomic: *φάσις* [24]
    - noun: *ὄνομα* broad sense [25]
      - noun: *ὄνομα* strict sense [26]
        - simple
        - composite [28]
        - individual
        - universal [27]
      - negated noun [29]
      - flexion of noun [30]
      - verb *ῥῆμα* [31]
      - negated verb [32]
      - flexion of verb [33]
    - other *φάσεις*
  - molecular: *λόγος* [34]
    - sentence: *ἀπόφανσις* [34]
      - atomic [35]
        - affirmation: *κατάφασις*
        - negation: *ἀπόφασις* [35]
        - with individual subject
        - with universal subject taken
          - universally
          - not universally [36]
      - molecular [35]
    - other *λόγοι* (also called *φάσεις*) [35]

An atomic *symbol* is one which has no parts significant *as* parts of it;[38] an atomic *sentence* has significant parts but they are not sentences.[39] Nouns and verbs with prefixed negation are called "indefinite" (*ἀόριστον*)[40], flections of both nouns and verbs are, in Aristotelian terminology, "cases"[41] and are considered as belonging to a distinct syntactical category. A sentence must be composed, according to the *De Int.*, of a noun or an infinite noun and of a verb or a case of a verb; "is" and "is not" (respectively in other tenses) seem to be considered in one text as being verbs;[42] but in another, Aristotle teaches that they mean only the com-

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[23] *De Int. 2, 16 a 19f.; 27ff.; 4, 17 a 1f.* — [24] *De Int. 5, 17 a 17ff.* — [25] *De Int. 3, 16 b 19ff.* — [26] *De Int. 2, 16 a 19ff.* — [27] *De Int. 7, 17 a 38ff.* — [28] *De Int. 2, 16 a 22ff.; 4, 16 b, 32f.* — [29] *De Int. 2, 16 a 30ff.* — [30] *De Int. 2, 16 a 32f.* — [31] *De Int. 3, 16 b 6ff.* — [32] *De Int. 3, 16 b 11ff.* — [33] *De Int. 3, 16 b 16ff.* — [34] *De Int. 4, 16 b 26ff.* — [35] *De Int. 6, 17 a 8f.* — [36] *De Int. 5, 17 a 37f.* — [38] *De Int. 2, 16 a 21; 16 b 6; 4, 16 b 30ff.* — [39] *De Int. 4, 16 b 27.* — [40] *De Int. 2, 16 a 30ff.; 3, 16 b 11ff.;* also *10, 20 a 6f.* — [41] *De Int. 2, 16 a 32f.; 3, 16 b 16ff.* — [42] *De Int. 5, 17 a 9f.*

position (*σύνθεσιν*) and do not have a designatum.[43] In the *De Int.*, the copula is not necessarily required in a sentence,[44] but the sentences examined in the *An. Pr.* are always composed of two nouns and the copula "belongs to" (*ὑπάρχει τῷ*) which takes the place of the "is". Finally in *An. Pr. A 36* Aristotle says explicitly that the copula must not always be the "belongs to" and gives instances of such other copulae (cp. 11E).

5 D. Semantics

The explicit semantic scheme of Aristotle is a rather simple one: written words are symbols of spoken words, these are symbols of mental experiences, and mental experiences are again symbols of things;[45] thus spoken words are also symbols of things.[46] But the Aristotelian semantics is by far more complex, and so complex indeed that its explanation would require a long ontological and epistemological introduction which, of course, cannot be given here. We mention only that there is undoubtedly in Aristotle's doctrine something corresponding to the Stoic "*λεκτόν*" or to the late scholastic "*conceptus objectivus*": namely the "*λόγος*" which, while being highly ambiguous, acquires in some texts the unambiguous meaning "that which is meant by the word in opposition to the things themselves"[47]. This is, of course, no *λεκτόν* after the Stoic fashion, but conformably to the Aristotelian realism, an aspect of reality; it is, however, clearly distinguished from the concrete things themselves.

The meaning of the words is conventional. They seem to be divided into two classes: the elements of the first directly mean things, the other only "carry with them the meaning" (*προσσημαίνει*) of something, e.g. of the composition of the subject with the predicate (the words "is" and "is not")[48] or of the universal mode of the sentence (the words "all" and "none")[49].

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[43] *De Int. 3, 16 b 22ff.*, but cp. *5, 17 a 11f.* — [44] *De Int. 10, 20 a 3ff.* — [45] *De Int. 1, 16 a 3ff.* — [46] *Soph. El. 1, 165 a 7.* — [47] e.g. *Phys. Θ 8, 263 b 13*; *Met. A 10, 993 a 17*; *B 2, 996 b 8*; *Γ 7, 1012 a 23*; *N 1, 1087 b 12*; cp. *Bonitz 433 b 48ff.* — [48] *De Int. 3, 16 b 22ff.* — [49] *De Int. 10, 20 a 12ff.*

On the other hand words are divided into univocal (*συνώνυμα*) where the meaning (*λόγος*) is the same in several designata [50] — and ambiguous (*ὁμώνυμα, πολλαχῶς λεγόμενα*) where this is not the case [51]. The latter are subdivided into (1) accidentally ambiguous (*ἀπὸ τύχης*) [52] which are sometimes called *ὁμώνυμα* in a narrower sense [53] and (2) another class of words which we may call "systematically ambiguous". This is again subdivided in one text into classes of terms which are called *ἀφ' ἑνός, πρὸς ἕν* and *κατὰ ἀναλογίαν* ambiguous [54]. An instance of the *πρὸς ἕν* is "healthy": a thing is called "healthy" because it possesses health, or produces it, or is its symptom or possible subject [55]. The situation seems, consequently, to be this: the word a is called "*πρὸς ἕν* ambiguous" in respect of the things x and y and of the attributes φ and ψ, if and only if $\varphi x . \psi y$ and one of the above relations holds between φ and ψ, and a means both φ in x and ψ in y.

We do not find any so explicit explanation of analogically ambiguous words; Aristotle writes, however, often on analogy, [56] and some of those texts seem to refer to such words. The situation seems to be this: let be four objects a, b, c, d and two relations Q and R such that aQb and cRd and let Q be identical or similar to R; then a name meaning both $D'Q$ and $D'R$ will be called "analogically ambiguous".

A symbol must not be used ambiguously in proofs [57]; here accidental ambiguity must be meant, not systematical. On the other hand Aristotle insists on the unity of meaning of atomic symbols [58]; in one text [59] he says e.g. that if a symbol means something it cannot mean at the same time its negation.

We may still notice that the verb, [31] but not the noun, [26] carries with it the meaning of time. A further investigation into Aristotle's semantics would lead us into an exposition of his theory of definition [60] which requires, in order to be understood, a rather

[50] *Cat. 1, 1 a 7*, but confirmed by several genuine texts, cp. *Bonitz 734 b*, e.g. *Soph. El. 19, 177 a 9ff.* — [51] *Cat. 1, 1 a 1ff.*; *Top. Z 10, 148 a 24*, cp. *Bonitz 737 b.* — [52] *Eth. Nic. A 4, 1096 b 26.* — [53] *Met. Γ 2, 1003 a 33.* — [54] *Eth. Nic. A 4, 1096 b 27f.* — [55] *Met. Γ 2, 1003 a 35ff.* — [56] cp. Muskens. — [57] *Soph. El. 4, 165 b 30ff.* — [58] *De Int. 11, 20 b 13ff.* — [59] *Met. Γ 4, 1006 b11 ff.* — [60] Bibliography of the subject in Philippe nos. *11. 61ff.*

extensive knowledge of his ontological and epistemological doctrines: it must be omitted here. We shall, however, briefly deal with the Aristotelian theory of truth.

5 E. Truth

Aristotle recognizes that "is" and "is not" are sometimes used as meaning "is true" resp. "is false" [61] but he emphatically distinguishes both [62]. Truth and falsehood are not attributes of things, but of thoughts [63] and of *λόγοι*; the latter expression carries with it its indeterminate status in the Analytics — but means clearly spoken words in the *De Int.* — The definition of truth, explicitly given as such, is: "to say that what is is and what is not is not is true" and conversely for falsehood [64]. Thus facts are the reason of the truth of opinions [65]. Among thoughts only composite ones are true or false [66]; among symbols, sentences only [67]; Aristotle seems to give as reason of this that only sentences have facts (the 'to be' or 'to be not') as meanings, while other symbols simply mean things [68]. The capacity of being true or false is even the characteristic of a sentence [69].

The relation between sentences meaning facts and those asserting the truth of the former is, according to Aristotle, one of mutual entailment; any way we have:

5. 1. $Ax \supset T\ulcorner Ax \urcorner$

5. 2. $T\ulcorner Ax \urcorner \supset Ax$ [70]

and, consequently, the syntactical properties of both are explicitely stated to be identical [71]. Let us note still that Aristotle distinguishes "wholly false" and "not wholly false" [72]; the former is the contrary of a true sentence, the latter a sentence such that its contradictory, but not its contrary is true.

On the other hand we find nothing in Aristotle's works relating to the distinction of entailment and implication, nor between the different meanings "truths" may have.

[61] *Met. Δ 7, 1017 a 31f.* — [62] E.g. *Met. Γ 6, 1011 b 15ff.* — [63] *Met. E 4, 1027 b 25ff.* — [64] *Met. Γ 7, 1011 b 26ff.* cp. *1012 a 3.* — [65] *Met. Θ 10, 1051 b 6ff.* — [66] *De An. Γ 8, 432 a 11ff.* cp. *6, 430 a 27ff.* — [67] *De Int. 4, 17 a 1ff.* — [68] *De Int. 4, 16 b 28ff.* — [69] *De Int. 4, 17 a 2ff.* — [70] *De Int. 8, 18 a 40ff.* — [71] *An. Pr. A 46, 52 a 32f.* — [72] *An. Pr. B 2, 54 a 4ff.*

6. Topics

The Topics represent an early but important stage in the development of Aristotle's logic. The strictly formal theories will be treated later on (ch. 7 and 11); but we shall deal here with the general character of that work (A), the predicables (B) and categories (C) which form the Aristotelian theory of predication, and finally with sophistics (D).

6 A. Characteristics

The aim of the Topics is to teach a method by which "we shall be able to reason (*συλλογίζεσθαι*) from opinions that are generally accepted about every problem propounded to us" [1]. In fact the book is far more a textbook of the practice of discussion than a logical work. Its main subject is the commonplaces (*τόποι*), a word difficult to translate: it means something like a general principle out of which arguments may be drawn for concrete cases. [2] Those principles are either logical rules — there are here few laws, contrary to what happens in the Analytics — or methodological recommendations, and even psychological remarks. Aristotle declares [3] — again contrarily to what he will say in the Analytics — that there is no single principle out of which the method could be developed: he does not yet know his analytical syllogism. On the other hand the Topics contain a number of formal rules and also some doctrines which are not to be found elsewhere, but which, as it seems, were not repudiated by Aristotle.

6 B. The predicables

While looking for a classification of problems, Aristotle was led to formulate a historically very important classification of the meanings of "is". There are two such classifications in the Topics, known respectively as that of predicables (*κατηγορούμενα*) and of categories. The former, which is considered first [4], is *not* a classi-

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[1] *Top. A 1, 100 a 1ff.* — [2] Cp., Scholz, *Geschichte 26.* — [3] *Top. A 6, 102 b 35f.* — [4] *Top. A 5, 101 b 37ff.*

fication of absolute terms, but of relations between a subject and a predicate. Such relations imply or do not imply, according to Aristotle, convertibility i.e. equivalence of terms, and essentiality of the predicate in regard to the subject. As there are four possible combinations of those two relations, we get four kinds of predicables: (1) definition (*ὅρος*), essential and convertible; (2) property (*ἴδιον*), non-essential and convertible; (3) genus (*γένος*) or differentia (*διαφορά*), essential and non-convertible; (4) accident (*συμβεβηκός*), non-essential and non-convertible. The classification is proved to be exhaustive [5] — which is actually the case.

The Topics examine rules concerning all predicables; but in doing so, Aristotle found that what applies to the accident applies also to the genus and not conversely; this is also the case respectively with the property and definition. Moreover, the logical rules concerning the accident are by far the most numerous in the Topics. Thus the relation of essentiality appeared as formally irrelevant; in fact, at the beginning of the *An. Pr.* Aristotle states this explicitly [6]. It does not mean that he repudiated later on all considerations about essential predication [7]; but there is no doubt that by classifying predicables he discovered and clearly conceived formal logic.

6 C. The Categories

The second classification of the meanings of the copula rises out of an attempt to give a most general classification of all objects [8]. Such most comprehensive classes are called "categories" (*κατηγορίαι*). In the Topics they are ten: Essence, Quantity, Quality, Relation, Time, Position, State, Activity, and Passivity [9]; but in most other

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[5] *Top. A 8, 103 b 6—19.* — [6] *An. Pr. A 1, 24a 25ff.* — [7] This has been, if not asserted, at least implied by Solmsen, *Entwicklung.* — [8] *Top. A 9, 103 b 22ff.* — [9] The translation (by Pickard-Cambridge in Ross, *Works*) is not quite accurate, but it is difficult to find better English terms; in the text we have: *τί ἐστι, ποσόν, ποιόν, πρός τι, ποῦ, ποτέ, κεῖσθαι, ἔχειν, ποιεῖν, πάσχειν* — i.e. what, how large, of what quality, in relation to something, where, when, to be situated, to have, to act, to suffer.

texts Aristotle substitutes "Substance" (*οὐσία*) for "Essence" [10] and in all except the Topics he omits Position and State [11]. This doctrine is essentially methodological [11], but with it two logically important points are raised by Aristotle. First, he writes in the Analytics that the copula ("belongs to") has as many meanings as there are categories [12]. Consequently the classification is not only one of objects, but above all one of the modes of predication; and in the light of this we must note as historically false the widespread opinion accrediting Aristotle with the knowledge of only one type of sentence, that of class-inclusion. Second, Aristotle teaches that "being" and "one" are not genera, i.e. that there is no all-embracing class. [13] The proof runs as follows: (1) for all A: if A is a genus, there is a B which is its difference; (2) for all A and B: if B is the difference of A, then A is not the genus of B. Suppose now that there is an all-embracing genus V; then, for all A, V would be a genus of A [by definition]; but, as V is a genus, it must have some differences, say B [by (1)]; now V cannot be a genus of B [by (2)]; consequently V is not the all-embracing genus and we get a contradiction. From the point of view of recent logic, that proof probably offends the rules the theory of types or of syntactical categories. Nevertheless, the doctrine of categories

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[10] The problem of the relation between *τί ἐστι* and *οὐσία* cannot be treated here; it is very complex and the text of Topics A 9 is particularly difficult. Cp. Trendellenburg, *46f.* — [11] Prantl (I, 207, note 356) collected 28 texts in which categories are enumerated. Out of them, however, 12 have at the end *καὶ τ'ἄλλα* i.e. are not meant to be complete enumerations. The same must be said of *166 b 10, 1004 a 30* (where *οὐσία* is missing) *178 a 4* (without *ποιόν*) *317 b 8, 319 a 9, 1045 b 32, 1069 a 19, 1054 a 4, 1029 b 23* (where we find no *πρός τι*), since those three are most certainly categories. In *1089 b 24, 1001 b 29, 1054 a 5* the "*πάθη*" seems to be a generic name for other categories. Thus we are left with six texts only:

Phys. A 7, 190 a 31: οὐσία, ποσόν, ποιόν, πρὸς ἕτερον;

Eth. Nic. A 4, 1096 a 23: τί, ποσόν, ποιόν, πρός τι, τόπος,. χρόνος;

An. Post. A 22, 83 a 21: τί ἐστιν, ποιόν, ποσόν, πρός τι, ποιοῦν, πάσχον, ποῦ, ποτέ;

Phys. E 1, 225 b 5: οὐσία, ποιότης, ποῦ, ποτέ, πρός τι, ποσόν, ποιεῖν, πάσχειν;

Met. Δ 7, 1017 a 25: τί ἐστι, ποιόν, ποσόν, πρός τι, ποιεῖν, πάσχειν, ποῦ, πότε.

[12] *An. Pr. A 37, 49 a 6—8*; cp. *Top. A 9, 103 b 20ff.* — [13] *Met. B 3, 998 b 22ff.*

is systematically important: it is an attempt to classify not only objects but also *types* of objects, and it includes an explicit rejection of the all-comprehending class.

The same result was reached again in 1908, after Aristotle's doctrine had been forgotten.

6 D. Sophistics

The last book of the Topics [14] contains a classification and an analysis of fallacies. Aristotle divides them into two groups: fallacies dependent on diction (*παρὰ τὴν λέξιν*), six in number, [15] and fallacies not dependent on diction (*ἔξω τῆς λέξεως*), seven in number. [16] The logically important content of his theory regarding the former is the following: words and phrases when repeated in the same argument must have (1) exactly the same form and (2) exactly the same meaning. [17] Among the fallacies of the second group the *Ignoratio elenchi* (*παρὰ τοῦ ἐλέγχου ἄγνοιαν*) [18] is considered as the chief one to which all others may be reduced: for every fallacy consists in assuming that something is an argument when it is not. [19] Two other fallacies of that group are interesting: the *petitio principii* (*παρὰ τὸ τὸ ἐν ἀρχῇ λαμβάνειν*) and the consequent (*παρὰ τὸ ἑπόμενον*). Discussing the former, Aristotle states that we are not allowed to assume what we intend to prove, thus excluding circular proofs [20]. The discussion of the latter contains the explicit rejection of

$$p \supset q . q . \supset . p$$

and, also of

$$p \supset q . \sim p . \supset . \sim q$$ [21]

as invalid.

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[14] *Top. I = Soph. El.* There is another, less comprehensive treatise on the same subject: *An. Pr. B 16—18.* — [15] *Soph. El. 4, 165 b 24ff.* — [16] *Soph. El. 4, 166 b 21ff.* — [17] *Soph. El. 5, 167 a 21ff.* — [18] *Soph. El. 5, 167 a 21ff.* — [19] *Soph. El. 6, 168 a 17ff.* — [20] *Soph. El. 5, 167 a 36ff.* — [21] *Soph. El. 28, 181a 27ff.* In spite of the example *167 b 6—8* Aristotle thinks here in terms of the logic of terms, cp. *Soph. El. 28, 181 a 22ff.*

7. Opposition. Principles of contradiction and of the excluded middle

We shall give here, after a historical survey of the problem of negation (A), a sketch of the later Aristotelian theory of opposition (B), of the principle of contradiction (C), and of the tertium non datur (D). The formal theorems concerning the opposition will be stated in the next chapter.

7 A. Historical survey; early doctrine

The situation with regard to negation and opposition which Aristotle met can be understood from his own early teaching as it appears in the Topics and *Met Γ*. It may be seen there that while those problems seemed very important (which is natural, considering that logic was then above all dialectics, i.e. a theory of discussion), at least three things were confused; (1) terms and sentences and, consequently, their negations, (2) quantified and non-quantified sentences, (3) negative sentences and negation of sentences. The very fact that Aristotle himself reasons in Met *Γ* according to the false principle

$$\sim Sap \supset SeP$$ [1]

suffices to show how great the difficulty must have been to get a clear theory on those points. However, Aristotle was able to overcome these initial confusions almost completely.

Out of his early doctrines we shall take one only for special attention, the theory of four opposites (*ἀντιθέσεις*). [2] These are historically important and by comparison with the later doctrine of opposition show the progress accomplished by Aristotle. They are: the relative (*πρός τι*, between *a* and *R'a*); the contrary (*ἐναντία*, as just-unjust); the opposition between privation and

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[1] *Met. Γ 4, 1007 b 19ff.*; *1008 a 2ff.*, *28ff.*, *30ff.*; *7, 1011 b 35ff.*; *1012 a 7ff.*; cp. Łukasiewicz, *Der Satz* and Salamucha, *100ff.* — [2] Top. *B 2, 109 b 18ff.*; *8, 113 b 15*; *Met. I 3 1054 a 23*; *4, 1055 a 38* etc.

possession (*κατὰ στέρησιν καὶ ἕξιν*: blindness and sight), and contradictory opposition (*κατ' ἀντίφασιν*: just — not-just). Even the last is formulated in such a way that it is not clear if we have to do with terms or with sentences, but the latter supposition is the more probable.

7 B. Later theory of opposition

Already in the *De Int.* Aristotle developed a far more formal (metalogical) doctrine of opposition. For each sentence there is one [3] and only one [4] denial, provided that the words with which it is composed are not ambiguous. [5] The denial of an affirmative sentence is a negative sentence concerning (*κατά*) the same subject and conversely. The negation must qualify the copula in sentences with individual names as subjects and in sentences whose subject is a class name, but are not quantified. If we apply the negation in a quantified sentence to the quantifier, we get its contradictory ($\ulcorner SaP \urcorner$ — $\ulcorner \sim SaP \urcorner$; $\ulcorner SeP \urcorner$ — $\ulcorner SiP \urcorner$; the name is first *ἀντιφατικῶς ἀντικειμέναι* [6] then *ἀντικειμέναι* in a narrower sense). [7] If, on the contrary, the negation qualifies the copula, we have a contrary sentence ($\ulcorner SaP \urcorner$ — $\ulcorner SeP \urcorner$: *ἐναντίαι* [8]). A similar doctrine is expounded in the *An. Pr.* where Aristotle distinguishes three kinds of "real" opposition: $\ulcorner SaP \urcorner$ — $\ulcorner SeP \urcorner$, $\ulcorner SaP \urcorner$ — $\ulcorner \sim SaP \urcorner$, $\ulcorner SeP \urcorner$ — $\ulcorner SiP \urcorner$ (i.e. six with the converses) [9] and one merely verbal, namely $\ulcorner SiP \urcorner$ — $\ulcorner SaP \urcorner$. [10] Out of two contradictories one must be true, the other false, [11] while the contraries cannot be both true [12], but the falsity of one does not entail the truth of the other [13]. The scheme representing all those theorems would be the following [14]:

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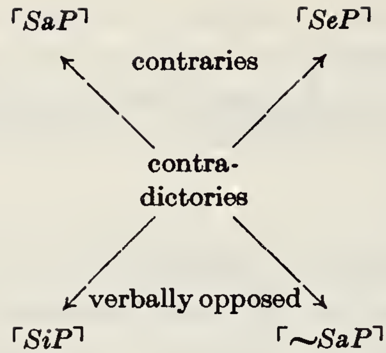
[3] *De Int. 6, 17 a 31f.* — [4] *De Int. 7, 17 b 38.* — [5] *De Int. 11, 20 b 14ff.* — [6] *De Int. 7, 17 b 16ff.* — [7] *De Int. 10, 20 a 30*; cp. *An. Pr. B 15, 63 b 28ff.*; *64 a 31f.* — [8] *De Int. 7, 17 b 20ff.*; cp. *An. Pr. B 15, 63 b 28*; *64 a 31.* — [9] *An. Pr. B 15, 63b 24ff.*; *64 a 33ff.*; *8, 59 b 8ff.* — [10] *An. Pr. B 15, 63 b 27* (cp., however, *8, 59 b 8ff.* — [11] *De Int. 7, 17 b, 26 f.* — [12] *De Int. 7, 17 b 22f.*; *10, 20 a 16ff.* — [13] *An. Pr. B 11, 61 b 6f.*; *62 a 17ff.* — [14] The "traditional" logical square is given first (among the texts preserved) by Apuleius (cp. 18 B).



Aristotle struggled hard with the initial confusion of the negation of a sentence with the negation of its terms; the chapter 10 of the *De Int.* in which he expounds the problem is one of the most instructive on the difficulties with which he had to deal. He does not seem to have reached a clear understanding of the different meanings of negation in both cases; but he elaborated a fragment of a correct theory of obversion [15]. Curiously enough, he never used it in his different axiomatizations of the syllogistic, where, as it has been shown recently [16], it would have been very useful. The formal theorems concerning that part of his teaching (as also the theory of opposition of modal sentences) will be treated later on.

7 C. The principle of contradiction

While we find no principle of identity in the preserved writings of Aristotle, [17] a whole book of Metaphysics (Γ) is devoted to the principle of contradiction (which is, however, not called so by Aristotle himself) and there are numerous formulations of it in his other works. Those formulations may be divided into logical and metalogical. The former are: "the same cannot belong and not belong together to the same under the same respect" [18] and "Let

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[15] *DeInt. 10, 20 a 20—26.* The importance of that text has been pointed out to the author by Fr. I. Thomas; cp. also his review of Sir W. D. Ross' Analytics in *Dominican Studies 3, 1950, 184ff.* — [16] Cp. I. Thomas, *CS*(n): *an extension of CS. Dominican Studies* 2, 1949, 145—160. — [17] The nearest approach to it is perhaps *An. Pr. A 32, 47 a 8f.*: *δεῖ γὰρ πᾶν τὸ ἀληθὲς αὐτὸ ἑαυτῷ ὁμολογούμενον εἶναι πάντῃ.* — [18] *Met. Γ 3, 1005 b 19f.*

A stand for 'to be good', B for 'not to be good' .... Then A and B will never belong to the same thing" [19]. This may be stated in the following terms:

$$7.11. \qquad (x, \varphi) . \sim (\varphi x . \sim \varphi x).$$

Among the metalogical formulations we have: "contradictory statements are not true together" [20] and "it is not possible to assert and deny the same" [21]. The latter was sometimes considered as a psychological law [22], but it may be doubted if it was meant to be one by Aristotle.

We may formulate it as follows:

$$7.12. \qquad \sim (T \ulcorner \varphi x \urcorner . T \ulcorner \sim \varphi x \urcorner)$$

but

$$7.13. \qquad \sim (T \ulcorner SaP \urcorner . T \ulcorner SoP \urcorner) . \sim (T \ulcorner SeP \urcorner . T \ulcorner SiP \urcorner)$$

would be probably more accurate.

There is also another form of the metalogical principle in which *contrary* sentences are said to be incompatible, either in the technical meaning of "contrary" or generically; this is stated both in its logical and in its metalogical form:

$$7.14. \qquad (x, \varphi) . \sim (\varphi x . \overline{\varphi} x) \text{ [23]}$$

$$7.15. \qquad (S, P) . \sim (T \ulcorner SaP \urcorner . T \ulcorner SeP \urcorner) \text{ [24]}$$

It might be noted that the principle of contradiction applies to actual attributes (*ἐντελέχεια*) only, since a thing may potentially (*δυνάμει*) have contradictory attributes. [25]

The principle is declared to be evident and "the firmest of all opinions" [26]. Aristotle demonstrates it elenchically, i.e. by reduction to absurdity. We shall not reproduce his arguments which seem to be all fallacious and which simply could not have been stated in

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[19] *An. Pr. A 46, 51 b 36ff.* — [20] *Met. Γ 6, 1011 b 15f.* — [21] *Met. Γ 3, 1005 b 23ff.* — [22] Łukasiewicz, *Der Satz, 17*; Salamucha, *75.* — [23] *Top. B 7,113 a 22f.* — [24] *De Int. 7, 17 b 20ff.; 10, 20 a 16ff.* — [25] *Met. Γ 5, 1009 a 35f.* — [26] *Met. Γ 4, 1005 b 19—23.*

that form later on, as they contain the *petitio principii* and other errors. [27] Another doctrine which was repudiated later is that this principle is the first axiom into which all demonstrations are reduced, since it is by nature the principle of all axioms (*φύσει ἀρχή*) [28]. In fact the opposite is thought in the *An. Post.*: it is explicitly said that no demonstration assumes that principle [29]. Moreover, both in the *An. Pr.* and in the *An. Post.* Aristotle gives instances of syllogisms which violate the principle of contradiction and yet are considered as perfectly valid, e.g.:

*7. 16. $SaM . SeM . \supset . SeS$ [30]

*7. 17. $Ma(P \cap -P) . SaM . \supset . Sa(P \cap -P)$ [31]

Similar instances are explicitly adduced in the *An. Post.* in order to show that our principle is not needed in any demonstration. The evolution of Aristotle in that regard is easy to understand: the principle of contradiction must have appeared as the foundation of deduction where the *reductio ad absurdum* was the main instrument of thought — as it was in dialectics. But when Aristotle discovered his non-dialectical, but positively logical doctrine, the logical importance of the principle must have been considerably diminished by it. This does not mean, however, that he ever doubted its validity.

7 D. The principle of excluded middle

On the other hand Aristotle seems to have seriously doubted the universal applicability of the *tertium non datur*. He distinguishes it clearly from the principle of contradiction and considers it in the Metaphysics [32] as a kind of corollary to this law. We have again several logical and metalogical formulations of this principle. The logical are: "there cannot be an intermediary between contradiction (*μεταξὺ ἀντιφάσεως*)" [33]. "Let A stand for 'to be good', B for 'not to be good' .... Then either A or B will belong to

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[27] cp. Łukasiewicz, *Der Satz, 21, 27ff.*; Salamucha. *100ff.* — [28] *Met. Γ 3, 1005 b 32ff.* — [29] *An. Post. A 11, 77 a 10ff.* — [30] *An. Pr. B 15, 64 a 1ff.* — [31] *An. Pr. B 15, 64 b 20f.* — [32] *Met. Γ 7, 1011 b 23ff.* — [33] L. cit., cp. *Top. Z 6, 143 b 15f.*

everything".[34] The "or" here has clearly the meaning of our "v" (matrix *"1110"*), for Aristotle states in the same phrase our 7. 11 as a distinct law. We may write

7. 21. $(x, \varphi).\varphi x \vee \sim \varphi x$

Among the metalogical formulations we have: "One of the two parts (*θάτερον μόριον*) of the contradiction must be true.... one of the two parts of the contradiction is false."[35] "Every affirmation is true or false".[36] All those formulae are metalogical, but the first one might perhaps be also interpreted as a logical law. We formulate them as follows:

7. 22. $T\ulcorner\varphi x\urcorner \vee T\ulcorner\sim\varphi x\urcorner . F\ulcorner\varphi x\urcorner \vee F\ulcorner\sim\varphi x\urcorner$

7. 23. $T\ulcorner\varphi x\urcorner \vee F\ulcorner\varphi x\urcorner$.

The distinction between 7. 21 and 7. 23 is clearly assumed in *De Int 9* where the former is deduced from the latter. But Aristotle does it in order to show that the assumption of the *tertium non datur* to individual, future, and contingent (*ἀπὸ τύχης*) events leads to absurd consequences. His reasoning is briefly this: if 7. 23, then 7. 21, but if so, one of the two, $\ulcorner\varphi x\urcorner$ or $\ulcorner\sim\varphi x\urcorner$ must be always true; and this implies that one of them is necessary; while it is evident, he says, that there are contingent events. Thus the application of 7. 21 to future contingent events is rejected. In the body of the Organon we find no trace of any consequence of those doubts, however. The *tertium non datur* is always supposed to be universally valid.

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[34] *An. Pr. A 46, 51 b 36ff.* — [35] *Met. Γ 8, 1012 b 10f.* — [36] *De Int. 9, 18 a 37f.*

8. Assertoric syllogistic. Description and methods

Aristotle's main and best known — if not always well understood — work in logic is his theory of the syllogism as explained in *An. Pr. A 4—6*. In order to avoid confusion of that doctrine with the theory of other principles also called "syllogisms", we shall term the formulae examined here "analytic syllogisms". They are divided into two classes: the assertoric (*τοῦ ὑπάρχειν*) and the modal syllogisms; the latter contain always at least one modal functor which is missing in the first class. [1]

We shall deal first with assertoric syllogisms, describing their structure (ch. 8) and stating the formal laws (ch. 9), then with modal syllogisms (ch. 10). In the present chapter there will be four paragraphs: on the fundamental definitions (A), the structure, meaning and import of syllogistic sentences (B), the three figures (C), and the methods of axiomatization (D).

8 A. Fundamental definitions

We find in the Organon no definition of the analytic syllogism; what is offered as such [2] is taken almost literally from the Topics and does not fit the practice. Aristotle gave, however, a thoroughgoing metalogical description of the syllogism [3] and out of his practice more details concerning its structure may be drawn. Thus we obtain the following description:

(1) The analytic syllogism is a substitution of $\ulcorner pq \supset r \urcorner$; this means that it is a conditional *sentence*; the formal principle of which it is a substitution (what will later on be called "mode") is a logical *law*, not a metalogical *rule*. [4]

(2) In the above $\ulcorner pq \supset r \urcorner$ each variable is substituted by an atomic sentence of the type $\ulcorner B$ belongs to $A \urcorner$ (*τὸ B ὑπάρχει τῷ A*) with or without quantification and negation. [5]

(3) The sentences substituted for "p" and "q", i.e. those the

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[1] Cp. chapter 10: *An. Pr. A 2, 25 a f.* — [2] *An. Pr. A 1, 24 b, 18—20.* — [3] *An. Pr. A 1, 23—26.* — [4] This has been re-discovered by Łukasiewicz, *Aussagenlogik.* — [5] *An. Pr. A 24, 42 a 32f.*; cp. *23, 40 b 36ff.*; *28, 44 b 6*; *B 2, 53 b 19*; *18, 66 a 17f.* — [6] *An. Pr. A 15, 35 a 11f.*, *31f.*; *A 18, 38 a 4f.*; *A 25, 42 b 9—10*; *An. Post A 19, 82 b 6f.*; *22, 84 a 35*; *23, 84 b 14*. *Διάστημα* (relation) is opposed to *πρότασις* *42 b 20*, but in most texts the terms are synonimous, especially in *An. Post*

product of which forms the antecedent, are called "premisses" (προτάσεις or διαστήματα [6]); the substitution for "r" is called "conclusion" (συμπέρασμα; often simply: συλλογισμός).

(4) What is substituted for "A" and "B" in the premisses and in the conclusion — indeed the letters themselves — are called "terms" (ὅροι). [7] They must be, according to Aristotle, three: [8] the first, in one of the premisses and in the conclusion; the second, in the other premiss and in the conclusion; the third in both premisses; this third is called "the middle" (μέσον), [9] both other terms are "the extremities" (ἄκρα). [10] The following is an instance: "if P belongs to all M, and all M belongs to all S, then P belongs to all S": "M" is here the middle term, "P" and "S" the extremities.

The sentences occurring in a syllogism are divided into affirmative and negative; and, according to the quantification, into universal, particular, and indeterminate (ἀδιόριστος); [11] the last are sentences without quantifier [12]. While examining them Aristotle found that they are equivalent to the corresponding particular sentences. [13] Consequently, we are left with only four types of sentences, namely: $\ulcorner SaP \urcorner$, $\ulcorner SeP \urcorner$, $\ulcorner SiP \urcorner$, and $\ulcorner SoP \urcorner$. Instead of $\ulcorner SoP \urcorner$, however, Aristotle says $\ulcorner \sim SaP \urcorner$. The subject only is quantified; the quantification of the predicate is emphatically rejected. [14]

Most of this is stated with the use of variables; in fact Aristotle develops here for the first time in history a system of formal logic laws. However, he himself does not distinguish the syllogism from what will later on be called a mode of a syllogism (τρόπος), i.e. from the formal law of which it is a substitution: he always speaks

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[7] *An. Pr. A 1, 24 b 16*; what was said above (ch. 5 A) must be remembered however: we do not know the semiotic status of the ὅρος. — [8] *An. Pr. A 25, 41 b 36ff.*; *28. 44 b 6f.*; *B 2, 53 b 19.* — [9] *An. Pr. A 4, 25 b 32—36*; *5, 26 b 36f.*; *6, 28 a 12f.*; *23, 41 a 2ff.*; *44 b 40ff.*; *31, 47 a 40ff.* etc.; *B 18, 66 a 28.* — [10] *An. Pr. A 4, 25 b 35ff.* etc. — [11] *An. Pr. A 2, 25 a 2—5.* — [12] *An. Pr. A 1, 24 a 19—22.* — [13] *An. Pr. A 4, 26 b 21ff.*; *5, 27 b 36—38*; *6, 29 a 8f.*; the equivalence is explicitly stated *A 7, 29 a 27ff.* — [14] *De Int. 7, 17 b 12ff.*; *An. Pr. A 27, 43 b 17—21.*

of syllogisms only. He discovered the variable: but his very text shows how the passage from a letter as shorthand for a name slowly changed into a variable; even so, it looks as if he never at all realized himself that he was dealing with variables.

8 B. Meaning and import of the sentences

There are two points concerning syllogistic sentences which have often been misunderstood and must be briefly mentioned here. First, when Aristotle converts those sentences in the practice of axiomatization of his syllogisms, he seems to take them in extension, i.e. as meaning class-inclusion; but if one considers what he himself says about the meanings of the copula (cp. 6C) it is not less evident that he does not mean all sentences to be taken in that way. There is in his writings no clear distinction of connotation and denotation, and while he takes the terms in the syllogistic axiomatization as if they meant classes, he most certainly would deny the reducibility of all sentences to that form. Moreover, both in his treatment of the modal syllogism [15] and in (later) considerations on assertoric syllogistics [16] Aristotle himself proposed the following intensional interpretation of the syllogistic sentences: "B belongs to all A" should mean, as it seems, either "A belongs to all of that to which B belongs" or "A belongs to all that to all of which B belongs". It will be easily seen that we could interpret those two formulae, respectively, by

$$(1) \qquad Bx \supset (x)Ax$$

$$(2) \qquad (x)Bx \supset (x)Ax$$

Furthermore, from the admission of several laws (9. 11; 12. 20, 23, 42, 59, 60, 64 and 67) it follows that all syllogistic sentences have an existential import. It was thought during the Middle Ages, and again in recent times, that those laws are false. Recent research has shown that this is not the case; only the functors (especially our "a") have a different meaning in the Analytics from that which is ascribed to them by recent logicians. In fact,

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[15] cp. *10 B.* — [16] *An. Pr. A 41, 49 b 14ff.*

the Aristotelian system has been correctly axiomatized on the basis of quite intuitive axioms, plus $\ulcorner AiA \urcorner$ which, precisely, states the existential import of all sentences.

8 C. The Three Figures

Another misunderstanding often met with concerns the divisions of analytical syllogisms into three figures (σχήματα).[17] If we use "M" for the middle term, "S" and "P" for the extremities, placing the predicate always after (to the right) of the subject (i.e. contrary to the Aristotelian use) those three schemes may be represented as follows:

$$(1)\quad \begin{array}{l} M - P \\ S - M \\ S - P \end{array} \qquad (2)\quad \begin{array}{l} P - M \\ S - M \\ S - P \end{array} \qquad (3)\quad \begin{array}{l} M - P \\ M - S \\ S - P. \end{array}$$

The question immediately occurs as to why Aristotle does not have four figures, the three already given, and:

$$(4)\quad \begin{array}{l} P - M \\ M - S \\ S - P \end{array}$$

As a matter of fact, he *has* (4) in the form of:

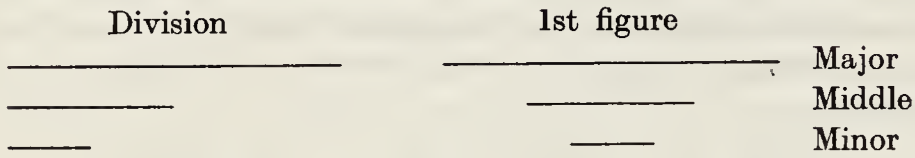
$$(1')\quad \begin{array}{l} M - P \\ S - M \\ P - S \end{array}$$ [18]

For if we interchange "S" and "P" in (1′) and consider the order of the premisses as irrelevant, we get precisely (4). But Aristotle did *not* consider (1′) as a distinct figure and examined the syllogism of the form of (1′) as if they belonged to (1).

The reason why he could do so is that he defined the two extremes, namely the major (μεῖζον) and the minor (ἔλαττον, ἔσχατον) not according to their formal position in the conclusion,

[17] First: *An. Pr. A 4, 25 b 32—35*; second: *5, 26 b 34—39*; third: *6, 28 a 10—15*; three only: *23, 41 a 16—18*. — [18] *An. Pr. A 7, 29 a 19—27*; *B 1, 53a 9—12*.

but to their extension; and this again becomes clear if we suppose, as in fact it is most probable, that his syllogism was developed out of the Platonic division (*διαίρεσις*) in such a way that the first figure was developed first of all and the two others formed by conversion of some of the functions in the modes of the first, at a later stage. The following scheme, in which the relative extension of the terms is indicated by lines, illustrates the point:



8 D. Axiomatization

The assertoric syllogism is probably the most important discovery in all the history of formal logic, for it is not only the first formal theory with variables, but it is also the first axiomatic system ever constructed. Aristotle's theory of the axiomatic system belongs to methodology and cannot be treated here; we shall limit ourselves to the remark that according to him there must be some undemonstrated axioms (*ἀξιώματα*) [19] while other theorems are deduced; that in each axiomatic system the number of steps must be finite [20]; and that, as far as *logical* axioms are concerned, they must be intuitively evident. [21] This theory has been, in fact, applied to syllogistics. Some modes are taken as axioms and out of them others are deduced. To do this, Aristotle uses three different methods; the direct reduction (*δεικτικῶς ἀνάγειν*), the reductio ad impossibile (*εἰς τὸ ἀδύνατον ἀνάγειν*), and the ecthesis (*ἐκτίθεσθαι*).

The direct reduction is based on the two rules analogous to the laws

[8. 11.] $$pq \supset r : \supset : s \supset p . \supset . sq \supset r$$

[8. 12.] $$pq \supset r : \supset : s \supset q . \supset . ps \supset r$$

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[19] *An. Post. A 3, 72 b 18ff.* — [20] *An. Post. A 19—20, 81 b 10ff.* — [21] *An. Post. B 19, 99 b 20ff.*

(which are never stated, however) and on the laws of the conversion which Aristotle stated and tried to axiomatize (9. 41—43). He proceeds as follows: given a mode of the form $\ulcorner pq \supset r \urcorner$ and a law of conversion $\ulcorner s \supset p \urcorner$ he gets a mode of the form $\ulcorner sq \supset r \urcorner$; he actually says: as $\ulcorner s \urcorner$ converts into $\ulcorner p \urcorner$, out of $\ulcorner sq \supset r \urcorner$ we get $\ulcorner pq \supset r \urcorner$.

The *reductio ad impossible* (contraposition) is based on rules analogous to the laws

[8. 21.] $$pq \supset r . \supset . \sim rq \supset \sim p$$
[8. 22.] $$pq \supset r . \supset . p \sim r \supset \sim q$$

which are stated explicitly in a generalized form (11. 63), and on the laws of the opposition (9. 20—31). Actually Aristotle proceeds as follows: a syllogism of the form $\ulcorner sq \supset t \urcorner$ is to be proved; suppose $\ulcorner sq \urcorner$ and deny $\ulcorner t \urcorner$; this negation is equivalent to $\ulcorner p \urcorner$; thus by multiplying the result by $\ulcorner q \urcorner$ we get $\ulcorner pq \urcorner$, i.e. the antecedent of a valid syllogism $\ulcorner pq \supset r \urcorner$; we get, consequently, $\ulcorner r \urcorner$; now this entails — by a law of the theory of opposition (9. 20ff) — $\ulcorner \sim s \urcorner$ i.e. the negation of the assumed $\ulcorner s \urcorner$. It is easy to see that Aristotle must have developed this method out of 3. 2 or a similar rule; he reached a more complex formula, however.

The third method is that of *ecthesis* (*ἔκθεσις*). From the extension of a term Aristotle "takes out" an individual and operates on its name in order to reach the desired result in the term of classes. The laws used here belong to the calculus of individuals and classes. The most important among them are: [22]

* 8. 31. $$\sim SeP . \supset . (\exists x) . x \,\varepsilon\, S . x \,\varepsilon\, P$$
* 8. 32 $$MaP . MaS . \supset . (\exists x) . x \,\varepsilon\, P . x \,\varepsilon\, S$$

In the following "*g*" is an individual name:

* 8. 33. $$g \,\varepsilon\, P . g \,\varepsilon\, S . \supset . g \,\varepsilon\, S . g \,\varepsilon\, P$$
* 8. 34. $$g \,\varepsilon\, S . g \,\varepsilon\, P . \supset . SiP$$
* 8. 35. $$g \,\varepsilon\, S . g \sim\varepsilon\, P . \supset . \sim SeP.$$ [23]

[22] It should be remembered that all syllogistic sentences have an existential import. — [23] The most important instances are: *An. Pr. A 2, 25 a 15ff.* (our 9. 41) and *An. Pr. A 6, 28 a 22ff.* (Darapti, i.e. 9. 59).

The above interpretation (8. 31—32) is not quite adequate, however, since Aristotle does not say "some of the A", but "takes out" a concrete individual. The leading idea is this: if there is an individual belonging to two classes then those two classes overlap.

Aristotle did not find a logical method which would allow one to show that a mode is false; he simply operated by substitutions, showing that in some cases the conclusion of such substitutions with true premisses is true, in some others false.

We may still note that in the later portions of the Analytics Aristotle tried to form a *metalogical* system of assertoric syllogisms. He described what must be the quantification etc. of premisses in each figure. This metalogical part of his system is not very important and is well known; it will be omitted here.

9. Formal laws of assertoric syllogistics

We shall be concerned here with the laws stated by Aristotle in his treatise of assertoric syllogism (as explained in the foregoing) and with them the theory of subalternation and of opposition which is constantly presupposed in that system. In order to simplify our exposition we shall also deal with some laws which are not used in syllogistics, namely those concerning the complementary class, which were considered in some books of the Organon as a part of the theory of opposition. There will be five paragraphs: on subalternation and syllogistic opposition (A), the complementary class (B), conversion (C), syllogistic proper (D) and the various axiomatic systems (E).

9 A. Subalternation and opposition

The laws of opposition, which were later incorporated into the so-called "logical square" are, indeed, stated by Aristotle, but curiously enough, are not developed as a part of his syllogistics — perhaps because he had considered the problems connected with them in previous works, *Top.* and *De Int.* But while the laws of opposition were at least restated in the Analytics and frequently alluded to, we find nothing of that kind in regard to the laws of subalternation. Which would have been:

[9. 11.] $SaP \supset SiP$

[9. 12.] $SeP \supset SoP$

with their converses

[9. 13.] $\sim SiP \supset \sim SaP$

[9. 14.] $\sim SoP \supset \sim SeP.$

Those laws are, however, easily deducible from the remaining, explicitly stated Aristotelian theorems. Moreover, we have in the Topics analoga of them in a vaguer form: 11. 13 (for 9. 11), 11. 14 (9. 12), 11. 11 (9. 13), 11. 12 (9. 14). Perhaps also the (later) so called "dictum de omni" ("we say that one term is predicated of all another whenever no instance of the subject can be found of which the other term cannot be asserted" [1]) might be considered as something equivalent to those laws — in spite of the fact it was stated by Aristotle as a definition.

The laws of opposition proper are frequently stated and used, but no attempt is made to axiomatize them. We have:

* 9. 20.	$SaP \supset \sim SeP$ [2]
* 9. 21.	$SaP \supset \sim SoP$ [3]
* 9. 22.	$\sim SaP \supset SoP$ [4]
* 9. 23.	$SeP \supset \sim SaP$ [5]
* 9. 24.	$SeP \supset \sim SiP$ [6]
* 9. 25.	$\sim SeP \supset SiP$ [7]
* 9. 26.	$SiP \supset \sim SeP$ [8]
* 9. 27.	$\sim SiP \supset SeP$ [9]
* 9. 28.	$SoP \supset \sim SaP$ [10]
* 9. 29.	$\sim SoP \supset SaP$ [11]

The following laws state the opposition between sentences containing the names of complementary classes:

* 9. 30.	$(x) \sim (x \varepsilon A . x \varepsilon - A)$ [12]
* 9. 31.	$(x) . x \varepsilon A \supset x \sim \varepsilon - A$ [13]

while

$$(x) . x \sim \varepsilon A \supset x \varepsilon - A$$ [14]

$$(x) . x \sim \varepsilon - A \supset x \varepsilon A$$ [15]

are rejected as false.

In the *De Int.* we find the following laws of obversion:

9. 32.	$Sa - P \supset SeP$ [16]
9. 33.	$SiP \supset \sim Sa - P$ [17]
9. 34.	$\sim x \varepsilon P \supset x \varepsilon - P$ [18]

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[1] *An. Pr. A 1, 24 b 28—29.* — [2] *An. Pr. B 14, 63 a 18ff., 23ff.* — [3] *An. Pr. B 12, 62 a 38f.* — [4] *An. Pr. B 11, 61 b 33f.* — [5] *An. Pr. A 2, 25 a 18f.; Top. B 1, 109 a 4—6.* — [6] *An. Pr. A 2, 25 a 21f.; B 14, 63 a 7ff.* — [7] *An. Pr. B 12, 62 a 34.* — [8] *An. Pr. B 14, 63 a 16f.* — [9] *An. Pr. B 13, 62 b 14f.* — [10] *An. Pr. B 11, 61 a 27—31.* — [11] *An. Pr. A 5, 27 a 39f.; B 11, 62 b 7f.* — [12] *An. Pr. A 46, 51 b 40.* — [13] *ib. 4—52 a 4.* — [14] *An. Pr. A 46, 52 a 4f.* — [15] *ib. 9.* — [16] *De Int. 10, 20 a 20f.* — [17] *De Int. 10, 20 a 22f.* — [18] *De Int. 10, 20 a 25f.*

9 B. Conversion

Aristotle says that a sentence of the form ⌜SxP⌝ (where the "x" stands for one of the functors "a" "e" "i" or "o") converts (ἀναστρέφει) if, when this sentence be assumed, another sentence of the form ⌜PxS⌝ (with a functor of the same or of a different form) must also be admitted. There are three laws of conversion of assertoric sentences:

* 9. 41. $SeP \supset PeS.$ [19]

This is proved as follows: suppose that 9. 41 is false. We then have ⌜SeP⌝ and ⌜PiS⌝. If so, there is at least one individual, say g such that $g \varepsilon P . g \varepsilon S$; by commutation (8. 33) we get $g \varepsilon S . g \varepsilon P$ and if so, we also have ⌜SiP⌝ which (by 9. 26) gives ⌜$\sim SeP$⌝, i.e. the negation of ⌜SeP⌝ which was supposed. The law supposed is ⌜$p \sim q \supset \sim p . \supset . p \supset q$⌝, but Aristotle does not state it. The rest is also less explicit than in our formulation, however all steps described above must have been more or less conscious.

* 9. 42. $SaP \supset PiS$ [20]

Proof: if not 9. 42, then we have ⌜SaP⌝ and ⌜$\sim PiS$⌝; this gives (by 9. 27) ⌜PeS⌝; out of which we obtain by 9. 41 ⌜SeP⌝ and further on, (by 9. 23) ⌜$\sim SaP$⌝ i.e. the negation of the ⌜SaP⌝ assumed. The law ⌜$\sim (p \supset q) \supset p \sim q$⌝ is used here.

* 9. 43. $SiP \supset PiS$ [21]

The proof is similar to the above, only 9. 24 is used instead of 9. 23.

It has sometimes been said that the proof of 9. 41 supposes 9. 43, but this is not the case. The central step is based on laws concerning individual names: 8. 31, 8. 32, 8. 35. These are different laws from 9. 43; only Aristotle uses the same kind of variables for classes and individuals, which caused the confusion. That confusion is not his; he distinguishes very clearly between laws concerning individuals and classes.

$$SoP \supset PoS$$

is rejected as false. [22]

[19] *An. Pr. A 2, 25 a 15; B 2, 53 a 11f.* — [20] *ib. 17f.; B 2, 53 a 10f.* — [21] *ib. 20f.; B 2, 53 a 10f.* — [22] *ib. 22—26; B 2, 53 a 12ff.*

9 C. Syllogisms

Aristotle stated first fourteen syllogistic laws (the later "modes"): [23]

First Figure:

* 9. 51. $MaP . SaM . \supset . SaP$ (Barbara) [24]
* 9. 52. $MeP . SaM . \supset . SeP$ (Celarent) [25]
* 9. 53. $MaP . SiM . \supset . SiP$ (Darii) [26]
* 9. 54. $MeP . SiM . \supset . SoP$ (Ferio) [27]

Second Figure:

* 9. 55. $PeM . SaM . \supset . SeP$ (Cesare) [28]
* 9. 56. $PaM . SeM . \supset . SeP$ (Camestres) [29]
* 9. 57. $PeM . SiM . \supset . SoP$ (Festino) [30]
* 9. 58. $PaM . SoM . \supset . SoP$ (Baroco) [31]

Third Figure:

* 9. 59. $MaP . MaS . \supset . SiP$ (Darapti) [32]
* 9. 60. $MeP . MaS . \supset . SoP$ (Felapton) [33]
* 9. 61. $MiP . MaS . \supset . SiP$ (Disamis) [34]
* 9. 62. $MaP . MiS . \supset . SiP$ (Datisi) [35]
* 9. 63. $MoP . MaS . \supset . SoP$ (Bocardo) [36]
* 9. 64. $MeP . MiS . \supset . SoP$ (Ferison). [37]

Later on he noted that in the first figure we may also have:

* 9. 65. $MaP . SeM . \supset . PoS$ (Fapesmo) [38]
* 9. 66. $MiP . SeM . \supset . PoS$ (Frisesomorum) [38]

and more similar laws by use of 9. 41—43. This last rule is not applied in detail by Aristotle, who states only three laws:

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[23] We give also the usual *scholastic* names (due to Peter of Spain). — [24] *An. Pr. A 4, 25 b 37ff.* — [25] *ib. 40ff.* — [26] *An. Pr. A 4, 26 a 23ff.* — [27] *ib. 25ff.* — [28] *An. Pr. 5, 27 a 5ff.* — [29] *ib. 9—14.* — [30] *ib. 32f.* — [31] *ib. 37f.* — [32] *An. Pr. A 6, 28 a 18f.*; *A 7, 29 a 37f.* — [33] *ib. 26ff.* — [34] *An. Pr. A 6, 28 b 7ff.*; actually: $\ulcorner MaS . MiP . \supset . SiP \urcorner$. — [35] *ib. 11*; actually: $\ulcorner MiS . MaP . \supset . SiP \urcorner$. — [36] *ib. 17ff.*; actually: $\ulcorner MaS . MoP . \supset . SoP \urcorner$. — [37] *ib. 33f.* — [38] *An. Pr. A 7, 29 a 23ff.*

* 9. 67.	$MaP.SaM.\supset.PiS$	(Baralipton)
* 9. 68.	$MaP.SiM.\supset.PiS$	(Dabitis)
* 9. 69.	$MeP.SaM.\supset.PeS$	(Celantes) [39]

It is worth while to note, however, that assuming the Aristotelian theory of the major and minor term (ch. 8C) it allows for the following laws:

[9. 70.]	$PeM.SaM.\supset.PeS$	(from Cesare) [40]
[9. 71.]	$PaM.SeM.\supset.PeS$	(from Camestres)
[9. 72.]	$MaP.MaS.\supset.PiS$	(from Darapti)
[9. 73.]	$MiP.MaS.\supset.PiS$	(from Disamis)
[9. 74.]	$MaP.MiS.\supset.PiS$	(from Datisi).

In the deduction of those laws the principle of syllogism $\ulcorner p\supset q:\supset:q\supset r.\supset.p\supset r\urcorner$ is used. The same principle with the laws of subalternation would allow deduction of the scholastic "subaltern" modes, five in number — *Barbari*, *Celaront*, *Cesaro*, *Camestrop*, and *Celantop* —, but there is no indication concerning them in the Organon.

9 D. Axiomatization

Aristotle developed his system axiomatically, at first in the following way: 9. 51—54 are assumed as axioms. They are said to be "perfect" (τέλειοι) syllogisms, i.e. such that "they do not need anything outside themselves in order to show their validity", [41] i.e. that they are intuitively evident. All others are proved: 9. 57 and 9. 63 by contraposition with use of 9. 51; the remaining eight by direct reduction: in the proof of 9. 55, 56, 65, 66, 9. 52

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[39] *An. Pr. B 1, 53 a 3—14.* — [40] It has been said sometimes — indeed I said it myself — that there is no difference between 9. 70 and 9. 56 from the Aristotelian point of view; but this is a mistake. In 9. 70 the major term is the subject of a negative premiss, while it is the subject of an affirmative premiss in 9. 56. No change of the position of premisses or renaming of the terms can influence this, as a term is a major or a minor term in Aristotle's syllogistic independently from its name and its position. Similar considerations apply to 9. 71—9. 55, 9. 73—9. 62 and 9. 74—9. 61. — [41] *An. Pr. A 1, 24 b 22ff.*

is used; 9. 59, 61 and 62 use 9. 53; 9. 58, 60 and 64 use 9. 54. To obtain 9. 56 and 9. 61, the order of the premisses must be changed and the conclusion converted.

Later, Aristotle discovered that two laws — 9. 51 and 9. 52 — are sufficient as axioms (with laws of the theory of conversion, opposition, and of, course, some rules of inference). He proceeds as follows: out of 9. 52 he gets 9. 55 and 9. 56 by conversion; then from 9. 56 he obtains 9. 53 and from 9. 55 he gets 9. 54. The rest is as in the first method. [42]

Still later he found that we may take the laws of whatever figure as axioms. [43] We shall not describe those various axiomatic systems in detail; the important thing which they show is that Aristotle seemed to have considered in a later stage all syllogistic laws as being on the same level and was only interested in their logical relations and deducibility. In *An. Pr. B* he proceeds quite as a contemporary logician would proceed.

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[42] *An. Pr. A 7, 29 b 1ff.* — [43] *An. Pr. A 45, 50 b 5ff.*

10. Modal Logic

The theory of modal sentences and syllogisms is the most developed and at the same time the most refined logical doctrine of Aristotle. It seems to be the last logical invention of the Logician, as it is both uncompleted in details and corresponds to Aristotle's own philosophy (which, as we know, deals not only with necessary facts as that of Plato, but with contingent ones). This theory, well-known and developed during the Middle Ages, was later almost completely misunderstood until A. Becker rediscovered its true meaning.

10 A. The modalities

Every sentence states, according to Aristotle, that somethings belongs, or necessarily belongs or may belong (to something other) [1]. Thus the modal functors do not qualify sentences, but the inherence, facts themselves. Each of the three modes of inherence is subdivided in the Organon into several kinds.

(1) *The necessity* (*τὸ ἐξ ἀνάγκης ὑπάρχειν*) may be divided first of all into ontological and logical: Aristotle says sometimes "it is necessary that *B* necessarily belongs to *A*" (*ἀνάγκη ὑπάρχειν ἐξ ἀνάγκης* [2] and even "it is necessary that *B* may belong to *A*" (*ἀνάγκη . . . ἐνδέχεσθαι ὑπάρχειν*) [3], where the first "necessary" means clearly purely logical necessity of entailment. On the other hand necessity is divided into absolute (*ἁπλῶς*) and hypothetical (*τούτων ὄντων*) [4]; the necessity of a fact, on the supposition that it *is* a fact (*ὅταν ᾖ*) [5], may be reduced to the latter.

(2) The assertoricity (*τὸ ἁπλῶς ὑπάρχειν, ὑπάρχειν μόνος*) is not, properly speaking, a modality, but it is nevertheless a mode of inherence. Aristotle has no technical term for assertoric sentences: he says simply "belongs only". But this "simple" belonging

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[1] *An. Pr. A 2, 25 a 1f.* — [2] *An. Pr. A 9, 30 a 39f.* — [3] *An. Pr. A 14, 33 a 26f.*; *15, 34 b 21f.* — [4] *An. Pr. A 10, 30 b 37—40*; cp. *13, 32 b 8—10*, also *Phys. B 9, 199 b 34ff*; *De Som. et vig., 455 b 26*; *De Part. an. A 1, 639 b 24ff.*; *642 a 9ff.* — [5] *De Int. 9, 19 a 23ff.*

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is again subdivided into absolute (ἁπλῶς) and temporal (κατὰ χρόνον), with different logical properties.[6]

(3) The *contingency* (τὸ ἐνδέχεσθαι ὑπάρχειν) offers the most complex problems. The two basic kinds of contingency[7] are the bilateral (E) which is sometimes called by Aristotle "contingency as defined" (scil. in *An. Pr. A 13*)[8] and the unilateral ($\Diamond$). We shall reserve the name of "contingent" for sentences containing the "ἐνδέχεται" taken in the former meaning and call those which contain it in the second meaning "possible". The (bilateral) contingent is defined as follows: "which is not necessary but, being assumed, results in nothing impossible",[9] i.e. a fact is contingent if and only if it is not necessary and not impossible. In symbols:

10. 11. $$E(Ax) . \equiv . \sim N(Ax) . \sim N(\sim Ax)$$

This is the meaning which is constantly used in *An. Pr. 8—22*, while in the *De Int.* Aristotle treats exclusively the possibility.[10] This is defined by:

10. 12. $$\Diamond (Ax) \equiv \sim N(\sim Ax).$$ [11]

The (bilateral) contingency is again subdivided into "what happens in most cases" (ὡς ἐπὶ τὸ πολύ) and into the indeterminate (ἀόριστον); but the text in which we find this distinction[12] is very confused. Yet this might have been a beginning of a logic of probability.

The relations between the above modalities may be stated in the following laws:

10. 13. $$N(Ax) \supset Ax$$ [13]
10. 14. $$Ax \supset \Diamond (Ax)$$ [14]
10. 15. $$N(Ax) \supset \Diamond (Ax)$$ [15]

[6] *An. Pr., A 15, 34 b 7—18.* — [7] *De Int. 13, 22 b 36f.; 23 a 7ff.* — [8] *An. Pr. A 15, 34 b 27f.; 17, 37 a 27f.* — [9] *An. Pr. A 13, 32 a 18—20*; cp. *3, 25 a 38ff.*; *De Int. 13, 22 b 36ff.*; and *Met. Θ 3. 1047 a 24ff.* — [10] *De Int. 13.* — [11] cp. 10. 31 below. — [12] *An. Pr. A 3, 25 b 14ff.* — [13] cp. *De Int. 13, 23 a 21f.* — [14] cp. *An. Pr. A 16, 36 a 15—17; 22, 40 a 25—32.* — [15] cp. *De Int. 13, 22 b 11.*

Those laws are stated by Aristotle himself — if in not a very sharp form. He also explicitly rejects as false:

$$N(Ax) \supset E(Ax)$$ [16]

and has there two special laws in addition:

* 10. 16.	$SeP \supset \Diamond (SeP)$ [17]
* 10. 17.	$SoP \supset \Diamond (SoP)$ [18]

Another law which results from the above definition is

[10. 18.]	$E(Ax) \supset \Diamond (Ax)$

while

$$Ax \supset E(Ax)$$

is invalid on the Aristotelian assumptions; but this is not stated by Aristotle himself.

10 B. The structure of modal sentences

In one — but in only one — text of the *An. Pr. A* [19] Aristotle describes a two-fold structure of the contingent sentences. He says that "*B* may belong to *A*" may mean either (1) *B* may belong to that to which *A* belongs or (2) *B* may belong to that to which *A* may belong. As this is, indeed, presupposed by most of the Aristotelian syllogisms and is, on the other hand, in accordance with his later analysis of the sentence [20], the Scholastics and recently A. Becker (but neither Theophrastus nor the ancient Commentators) understood the structure of modal sentences in two ways, which may be represented by the two following sets of laws.

(1)	10. 21.	$N(SaP) \equiv (x).Sx \supset N(Px)$
	10. 211.	$N(SeP) \equiv (x).Sx \supset N(\sim Px)$
	10. 212.	$N(SiP) \equiv (\exists x).Sx.N(Px)$
	10. 213.	$N(SoP) \equiv (\exists x).Sx.N(\sim Px)$

[16] *De Int. 13, 23 a 15f.* — [17] *An. Pr. A 16, 36 a 7—17.* — [18] *ib. cp. A 22, 40 a 25—32.* — [19] *An. Pr. A 13, 32 b 25—32.* — [20] *An. Pr. A 41, 49 b 14ff.*

(2) 10. 22. $N(SaP) \equiv (x).N(Sx) \supset N(Px)$
10. 221. $N(SeP) \equiv (x).N(Sx) \supset N(\sim Px)$
10. 222. $N(SiP) \equiv (\exists x).N(Sx).N(Px)\ x$
10. 223. $N(SoP) \equiv (\exists x).N(Sx).N(\sim P)$

Similar laws may be obtained from the above by substituting "E" for "N" (these will be referred to henceforth by the above number followed by "E").

Indeed, one of the most striking aspects of the Aristotelian modal syllogistic is that the principle "peiorem semper sequitur conclusio partem" which applies to assertoric syllogisms does not apply here. Thus we have e.g. (10. 512):

(1) $$N(MaP).SaM.\supset.N(SaP)$$

and also (10. 528):

(2) $$N(MeP).E(SaM).\supset.SeP.$$

Most of these laws become valid, indeed, if we assume 10. 21 resp. 10. 21 E, e.g. (1) becomes:

$$(x).Mx \supset N(Px) : (x).Sx \supset Mx : \supset : (x).Sx \supset N(Px)$$

which is a substitution of *Principia* * 10. 3. In some other laws 10. 22 E must be assumed, e.g. in 10. 514; this is

$$E(MaP).E(SaM).\supset.E(SaP)$$

and becomes with the use of 10. 22 E:

$$(x).E(Mx) \supset E(Px):(x).E(Sx) \supset E(Mx):\supset:(x)E(Sx) \supset E(Px).$$

The same structures are also presupposed by some of the laws of opposition and conversion stated below.

It is doubtful, however, that Aristotle had a clear idea of those structures when he wrote the bulk of his analysis. In fact, he does not mention them where they were most required (as in the justification of 10. 511 ff., 10. 512 ff. etc.). He also acknowledges as correct some laws of conversion which are manifestly invalid on those assumptions. E.g. in order to prove 10. 552 the major

$\ulcorner N(PeM)\urcorner$ is converted. Now this major must be interpreted here according to 10. 211, not to 10. 22; and if so, it is $\ulcorner(x).Px \supset \supset N(\sim Mx)\urcorner$ which, evidently, cannot be converted. It seems, consequently, that Aristotle was guided in the construction of his system by intuition only and that he first discovered these structures later on. Yet, in spite of the errors, his doctrine appears as a tremendous achievement.

10 C. The theory of negation and opposition

In *De Int.* Aristotle examined laboriously the logical relations between four modal functors: *δυνατόν, ἐνδεχόμενον, ἀδύνατον,* and *ἀναγκαῖον*; the first two are assumed to be equivalent and have the logical properties of our "$\Diamond$"; we shall consider them as one functor. The laws simplified in that manner are:

10. 31. $\Diamond(Ax) \equiv \sim I(Ax) \equiv \sim N(\sim Ax)$

10. 32. $\sim \Diamond(Ax) \equiv I(Ax) \equiv N(\sim Ax)$

10. 33. $\Diamond(\sim Ax) \equiv \sim I(\sim Ax) \equiv \sim N(Ax)$

10. 34. $\sim \Diamond(\sim Ax) \equiv I(\sim Ax) \equiv N(Ax)$; [21]

in one formula:

10. 35. $\Diamond(Ax) \vee I(Ax)$. [22]

In the Analytics the theory of negation of contingent sentences is expounded. Aristotle finds that

* 10. 36. $\sim E(SaP) \supset N(SiP) \vee N(SoP)$ [23]

which is correct, and moreover, shows that he reasons according to the so-called law of De Morgan

$$\sim (pq) \supset p \vee q.$$

Because of 10. 36 the method of the reductio ad impossible cannot be applied to many modal syllogisms.

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[21] *De Int. 13, 22 a 24ff.* — [22] *De Int. 13, 22 a 34ff.* — [23] *An. Pr. A 16, 37 a 24—26.*

Another important theory is stated in the following striking laws:

* 10. 37. $E(Ax) \equiv E(\sim Ax)$ [24]

* 10. 38. $E(SaP) \equiv E(SeP)$ [25]

* 10. 39. $E(SiP) \equiv E(SoP)$. [26]

That equivalence is meant here is clear from the use of these laws in the proofs of syllogistic modes. All of these laws are correct on Aristotelian assumptions, if we also presuppose the structure of modal sentences as explained above.

As a matter of fact we have:

$$\begin{aligned}
E(Ax) &\equiv \sim N(Ax) . \sim N(\sim Ax) && [10.\ 11] \\
&\equiv \sim N(\sim Ax) . \sim N(Ax) && [\text{commutation}] \\
&\equiv E(\sim Ax) && \left[10.\ 11\ \frac{\sim A}{A}\ \text{princ. of double neg.}\right] \\
E(SaP) &\equiv (x) . Sx \supset E(Px) && [10.\ 21\ \text{E}] \\
&\equiv (x) . Sx \supset . E(\sim Px) && [10.\ 37] \\
&\equiv E(SeP) && [10.\ 211\ \text{E}] \\
E(SiP) &\equiv (\exists x) . Sx . E(Px) && [10.\ 212\ \text{E}] \\
&\equiv (\exists x) . Sx : E(\sim Px) && [10.\ 37] \\
&\equiv E(SoP) && [10.\ 213\ \text{E}].
\end{aligned}$$

10 D. Conversion

The laws of conversion of necessary and possible ($\Diamond$) sentences are analogous to 9. 41—43:

* 10. 41. $N(SeP) \supset N(PeS)$ [27]

* 10. 42. $N(SaP(\supset N(PiS)$ [28]

* 10. 43. $N(SiP) \supset N(PiS)$ [29]

* 10. 44. $\Diamond(SeP) \supset \Diamond(PeS)$ [30]

* 10. 45. $\Diamond(SaP) \supset \Diamond(PiS)$ [31]

* 10. 46. $\Diamond(SiP) \supset \Diamond(PiS)$. [32]

It may be remarked that 10. 41—43 meet a serious difficulty if the structure 10. 21 ff. is presupposed.

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[24] *An. Pr. A 13, 32 a 37f.* — [25] *ib. 38f.* — [26] *ib. 40.* — [27] *An. Pr. A 3, 25 a 29ff.* — [28] *An. Pr. A 3, 25 a 32ff.* — [29] *ib.* — [30] *An. Pr. A 3, 25 b 3ff.* — [31] *An. Pr. A 3, 25 a 40—b 2.* — [32] *ib.*

The laws governing the conversion of contingent (E) sentences are different:

$$E(SeP) \supset E(PeS)\ ^{33}$$

is rejected as invalid, and, indeed, it must be, for if it would be admitted,

$$E(SaP) \supset E(PaS)$$

would also be in the system (because of 10. 38). On the contrary, we have:

* 10. 47.	$E(SoP) \supset E(PoS)$ [34]
* 10. 48.	$E(SaP) \supset E(PiS)$ [35]
* 10. 49.	$E(SiP) \supset E(PiS)$. [36]

The text in which these laws are stated is very confused. The proofs are bad and instead of our "E" we have there the "ἐπὶ τὸ πολύ" which, evidently, has different logical properties; but the above laws are constantly used (excepted 10. 47). As to their validity we may remark that $\ulcorner E(SoP) \urcorner$ is equivalent (by 10. 39) to $\ulcorner E(SiP) \urcorner$ and this — if the 10. 222 E is assumed — may be converted without changing its value — but not so if we assume 10. 212 E. The same is true about 10. 48.

Aristotle did not notice that on the same assumptions we could have

$$E(SeP) \supset E(PoS);$$

likewise, he never used 10. 47, which could have been useful in the proofs of the modal laws analogous to Baroco and Bocardo.

10 E. Syllogisms

The modal syllogistic of Aristotle is developed in the same fashion as his assertoric system. The laws it contains may be divided into three main classes (1) primary laws, analogous to some of the 9. 51—64, 95 in number; (2) laws obtained by means of

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[33] *An. Pr. A 3, 25 b 16f.* — [34] *An. Pr. A 3, 25 b 17f.* — [35] *An. Pr. A 3, 25 a 40—b 2.* — [36] *ib.* — [37] *An. Pr. A 15, 34 a 29ff.*

10. 14 from one of the first class, 7 in number; (3) laws obtained from them by means of 10. 38 or 10. 39, 35 in number. Together we have 137 laws. As premisses, only those with "*N*", "*E*", and without a modal functor (denoted here by "*Y*") are taken into consideration — thus not those with "$\langle\rangle$". All (8) combinations of those premisses are studied. The laws of the first figure are accepted as axioms in all combinations, except the sixth and the eighth (*NE*). The remaining are deduced from them by methods identical with those used in assertoric syllogistics, most by conversion; the reductio ad absurdum is used in order to prove the modes of the first figure of the eighth group (*NE*) and of the analogon of *Bocardo* in the fifth (*EY*), and ecthesis in the proofs of the analoga of *Baroco* and *Bocardo* in the first group. There is no proof for the analoga of the same modes in the second and third group (*NY* and *YN*) — in spite of the fact that it would have been easy to deduce them from 9. 58 and 9. 63 (using 8. 11 or 8. 12). The most complex problem is the proof of the modes of the sixth group (*YE*). As $\ulcorner Ep \supset p \urcorner$ is false in Aristotle's system, those modes cannot be recognized as intuitively valid; nor can they be proved by reductio ad impossible, for the negation of contingent premisses yields an alternative (cp. 10. 36) which is not considered by Aristotle as a possibile premiss of a syllogism. He proceeds as follows: he substitutes an assertoric sentence to the *E*-premiss and then proceeds by reductio ad impossibile. The process has been shown to be erroneous by Becker and we need not explain it here; the only thing which is worthy of note is an explicit formulation of a law belonging to the logic of propositions, namely 11. 64.

The following table shows the laws of the first class with a short indication of the laws of conversion used and of the modes out of which they are deduced.

11. Non-analytical laws and rules

We find in the Organon, along with the "analytic" laws, i.e. modes of syllogisms and the theorems concerning conversion and opposition, also about 60 formulae which are, at least partly, recognized by Aristotle as syllogisms, but not as syllogisms of the analytical type; we shall call them "non-analytic" and deal with them here. After a short introduction, (A), we shall state the laws belonging to the logic of predicates and classes (B), syllogisms based on hypothesis (C), the theory of identity (D), laws belonging to the logic of relations (E), and of propositions (F).

11 A. Historical introduction

Modern commentators of Aristotle were fascinated by the Aristotelian syllogistics to an extent that they often overlooked the wealth of non-analytical formulae which the Organon contains. Those formulae are most certainly recognized by Aristotle himself, also in his last period, as valid formal rules or laws. He says explicitly that not all logical entailment is "syllogistic" (which means here "analytic") [1]; he enumerates several kinds of syllogisms "based on hypothesis" [2] and promises to examine them [3]; he also declares that one cannot reduce (*ἀνάγειν*) such syllogisms to analytical laws [4]. Most certainly all attempts to do away with those laws or to reduce them to the Barbara-Celarent are un-Aristotelian. And yet, there is some basis for such attempts in Aristotle himself, for in his later period he considered the non-analytical syllogisms as being of lesser dignity and asserted many times that they do not "demonstrate" [5]. This we may understand if we remember that demonstration (*ἀπόδειξις*) always proves the necessary inherence of a property [6], which, evidently, can be done only by an analytical syllogism of the first figure [7]. Nevertheless, there is

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[1] *An. Pr. A 32, 47 a 22ff*; this as compared with the definition of the syllogism *Top. A 1, 100 a 25f.* and *An. Pr. A 1, 24 b 18f.* shows the shift in the meaning of *συλλογισμός*. — [2] *An. Pr. A 28, 45 b 16f.* — [3] *An. Pr. A 28, 45 b 15—20.* — [4] *An. Pr. A 44, 50 a 16f.* — [5] *An. Pr. A 44, 5 a 24.* — [6] *An. Post. A 6, 75 a 12ff.* — [7] *An. Post. A 14, 79 a 23ff*, cp. *24—25.*

no doubt that Aristotle recognized other laws as perfectly valid.

From the genetic point of view such formulae fall into three classes. (1) First there are laws and rules elaborated in the Topics before Aristotle discovered his analytic syllogism; some of them were re-examined and stated with variables in the later portions of the Prior Analytics — thus there is no reason to suppose that Aristotle rejected them later on as invalid; moreover, the fact that Theophrastus seems to have commented on them shows that even in his last period Aristotle acknowledged their validity. (2) Then we have some laws which Aristotle himself considered (erroneously) as being analytical, namely the (later) so-called "modes of the oblique syllogisms". (3) Finally there are several rules and laws manifestly discovered in the process of examination of the analytic syllogistics. As we have said this was done very thoroughly and led to important discoveries; among others, to the discovery of some laws with propositional variables.

11 B. Logic of classes and predicates

We have several such laws in the Topics:

11. 11. $A \subset B . \supset . - B \subset - A$ [8]

11. 12. $- A \subset - B . \supset . B \subset A$ [9]

But

$$A \subset B . \supset . B \subset A$$

is rejected as invalid [10].

11. 13. $(x)Ax \supset (\exists x)Ax$ [11]

11. 14. $(x) \sim Ax \supset \sim (x)Ax$ [12]

11. 15. $(x)Ax \supset \sim (\exists x) \sim Ax$ [13]

11. 16. $(x)Ax \supset \sim [(\exists x)Ax . (\exists x) \sim Ax]$ [14]

11. 17. $(x) \sim Ax \supset \sim [(\exists x)Ax . (\exists x) \sim Ax]$ [15]

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[8] *Top. B 8, 113 b 17f.*; cp. *4, 124 b 7ff.* — [9] *Top. B 8, 113 b 23f.* — [10] *ib. 20.* — [11] *Top. B 2, 109 a 3f.*; cp. *Γ 6, 119 a 35f.*; *120 a 15ff.* — [12] *Top. B 2, 109 a 4ff.*; *Γ 6, 120 a 8—10*; *119 a 36.* — [13] *Top. Γ 6, 120 a 8—10.* — [14] *ib. 21.* — [15] *ib.*

There is also in the Topics a number of laws concerning the contraries:

11. 21.	$A \subset B . \supset . \bar{B} \subset \bar{A}$ [16]
11. 22.	$(x) Ax \supset (x) \bar{A}\bar{x}$ [17]
11. 23.	$(\exists x) \sim Ax \supset (\exists x) \sim \bar{A}\bar{x}$ [18]
11. 24.	$(\exists x) A\bar{x} \supset (\exists x) \bar{A}x$ [19]
11. 25.	$(\exists x) \bar{A}x \supset (\exists x) A\bar{x}$ [20]

At the time he wrote his Analytics it may be doubted if Aristotle would have recognized 11. 21—25 as valid laws; in any case, at that time he never considered contrariety in his logical formulae.

11 C. Syllogisms based on hypothesis

We shall gather here rules and laws explicitly called such by Aristotle (*ἐξ ὑποθέσεως*) and, with them, those rules and laws which either seem to belong to the same class or were called "hypothetical syllogisms" by the commentators of the Organon. Most of these are, or correspond to, rules of inference which are of very frequent use both in everyday life and in science. Some were already known, as we have seen, by the forerunners of Aristotle.

Aristotle's own description of the syllogisms based on hypothesis is difficult to understand. We do not have his treatise on that subject which he promised to write, and the text in which he touches upon it in the most explicit manner [21] is either corrupted, or (which is more probable) was hastily written and contains logical errors. Aristotle examines there a substitution of the formula $\ulcorner \sim Ax \supset \sim Bx . \sim Ax . \supset . \sim Bx \urcorner$. He says that "$\sim Aa \supset \supset \sim Ba$" is not proved, but assumed, (*ἐξ ὑποθέσεως*), while "$\sim Aa$" is proved; then he goes on demonstrating this "$\sim Aa$" by syllogisms, in a confused and erroneous way. He concludes by saying that one must admit "$\sim Ba$" (*ὁμολογεῖν ἀναγκαῖον*). But this has not been demonstrated; it is assumed "ex hypothesi". He says, however, that we must agree to the conclusion, and as far as the $\ulcorner \sim Ax \supset \sim Bx \urcorner$ is concerned, he admits that in some cases not

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[16] *Top. B 8, 113 b 34f.* — [17] *Top. Γ 6, 119 a 39f.* — [18] *Top. Γ 6, 119 b 1f.* — [19] *ib. 4.* — [20] *ib. 5.* — [21] *An. Pr. A 44, 50 a 19—26.*

even this must be explicitly postulated, as it is "evident" (φανερόν) [22]. Thus, there is no doubt that he recognized the *modus ponens*, which is the rule used here, as valid; he only says — quite correctly from his (methodological) point of view — that such a syllogism does not "demonstrate" (ἀποδείκνυται). The fact that he rejected the possibility of reduction of such rules to analytical laws shows that he was well conscious of their particular logical nature.

11. 31. $$Ax \supset Bx . Ax . \supset . Bx.$$ [23]

More exactly Aristotle has here

$$\sim Ax \supset \sim Bx . \sim Ax . \supset . \sim Bx$$

but 11. 31 is supported by a text of the *Soph. El.* [24] The substitution given there is such that one might even think of a formula of logic of propositions; this seems excluded, however, by a statement of Aristotle in the same work. [25]

$$Ax \supset Bx . \supset . Bx \supset Ax$$

is explicitly rejected. [26]

11. 32. $$Ax \supset Bx . \supset . \sim Bx \supset \sim Ax.$$ [27]

This is an analogon of 11. 11; the variables are quite clearly predicate-variables, not propositional variables, as it appears from the comparison with another text [28] where the same expression ⌜if A is⌝ (τοῦ A ὄντος) is explained by substitutions. This is important for the understanding of the formulae stated by the commentators.

11. 33. $$Ax \supset Bx . Bx \supset Cx . \supset . Ax \supset Cx.$$ [29]

There again, we have a law of the logic of predicates, not of propositions.

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[22] *An. Pr. A 44, 50 a 35ff.* — [23] *An. Pr. A 44, 50 a 19ff.* — [24] *Soph. El. 5, 167 b 6ff.* — [25] *Soph. El. 28, 181 a 22—30.* — [26] *Soph. El. 5, 167 b 2f.* — [27] *An. Pr. B 4, 57 b 1—2.* — [28] *An. Pr. A 32, 47 a 28ff.* — [29] *ib.*

11. 34. $Ax \vee Bx . Ax . \supset . \sim Bx$
11. 341. $Ax \vee Bx . Bx . \supset . \sim Ax$
11. 342. $Ax \vee Bx . \sim Ax . \supset . Bx$
11. 343. $Ax \vee Bx . \sim Bx . \supset . Ax .$ [30]

Exclusive alternative is meant. In all the above laws no mention of the quantifier is made. On the contrary, we find explicit quantification and non-exclusive alternative in the two following, remarkable laws:

11. 351. $(x) . \sim (Ax . Bx) : (x) . Ax \text{ v } Bx : (x) . \sim (Cx . Dx) : (x) . Cx \text{ v } Dx : (x) . Cx \supset Ax : \supset : (x) . Bx \supset Dx$
11. 352. [Same Hyp.] $\supset : (x) \sim (Bx . Cx) .$ [31]

The hypothesis could be abbreviated as

$$(x) . Ax \vee Bx : (x) . Cx \vee Dx : (x) . Cx \supset Ax,$$

but Aristotle states it in the above form. In the proof of these two laws he uses, as it seems, quite consciously:

[11. 37.] $(x) : \sim (Ax . Bx) . Bx . \supset . \sim Ax$
[11. 38.] $(x) : Ax \supset Bx . \sim Bx . \supset . \sim Ax$
[11. 39.] $(x) : Ax \text{ v } Bx . \sim Ax . \supset . Bx .$

Because of lack of space we cannot reproduce the proof, which is remarkable.

11 D. Theory of identity

In *Top. H* Aristotle develops a theory of identity which contains the following laws:

11. 41. $x = z . y \neq z . \supset . x \neq y$ [32]
11. 42. $x = y . \supset . (A) . Ax \supset Ay$ [33]
11. 43. $A = B . \supset . (x) . Ax \supset Bx .$ [34]

The following two laws are only indicated:

[11. 44.] $\sim (A) . Ax \supset Ay . \supset . x \neq y$ [35]
[11. 45.] $\sim (x) . Ax \supset Bx . \supset . A \neq B$ [36]

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[30] *Top. B 5, 112 a 24—30.* — [31] *An. Pr. A 46, 52 a 39ff.* — [32] *Top. H 1, 152 a 31f.* — [33] *ib. 34f.* — [34] *ib. 35f.* — [35] *ib. 36f.* — [36] *ib.*

These laws form one half of the Leibnizian principle (more exactly, of two principles, of which — it is interesting to note — only one appears in the *Principia*, where we find no analogon of 11. 43). The other half of 11. 42 is also stated by Aristotle in another text as:

11. 46. $$x \neq y . \supset . \sim (A) . Ax \supset Ay$$ [37],

but the formula is rather vague.

Curiously enough, we do not find in the Organon the *logical* principle

$$x = y . y = z . \supset . x = z$$

A mathematical analogon (with "ἴσος", "equal") is well known to Aristotle.

11 E. Logic of relations

Contrary to what is often said, Aristotle knew a number of laws belonging to the logic of relations.

11. 51. $$A \subset B . \supset . R\text{``}A \subset R\text{``}B.$$ [38]

An alternative interpretation would yield

$$Q \subset R . \supset . D\text{`}Q \subset D\text{`}R,$$

but 11. 51 seems more correct. De Morgan is reported [39] to have said that the whole of Aristotle's logic could not prove that, because the horse is an animal, the head of the horse is the head of an animal. The authors of the *Principia* pointed out that this was a merit of Aristotle's logic, since the proposed inference is fallacious without an added existential premiss. It is, however, rather amusing to see that a similar *correct* law *is* to be found in Aristotle: this is precisely 11. 51. [40]

11. 52. $$A \subset -B . \supset . \breve{R}\text{``}A \subset -(\breve{R}\text{``}B).$$ [41]

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[37] *Soph. El..24, 179 a 37—39.* This has been discovered and pointed out to the author by Fr. I. Thomas O.P. From this we get, namely, by contraposition $\ulcorner (A) . Ax \supset Ay . \supset . x = y \urcorner$. — [38] *Top. B 8, 114 a 18f.*; cp. *Γ 6, 119 b 3f.* and also *B 10, 114 b 40ff., 115 a 1f.* This was discovered independently from the author by K. Dürr. — [39] *Principia I, 291,* ad * *37. 62.* — [40] 11. 51 is *Principia* * *37. 2.* — [41] *Top. B 8, 114 a 24f.*

In the chapter devoted to the (later) so-called "oblique" syllogism Aristotle states explicitly that the "belongs to" in a premiss of a syllogism may be substituted by another relation. He states three laws of such kind:

11. 53. $Q \in R . A \subset D'Q . \supset . A \subset D'R$ [42]
11. 54. $A \cup B \subset D'R . C \subset A \cap B . \supset . C \subset D'R.$ [43]

It is a remarkable theorem because it presupposes

$$A \cap B \supset A \cup B,$$

which is, in fact, a correct law.

11. 55. $D'Q \subset D'R . A \subset D'Q . \supset . A \subset D'R.$ [44]

Finally we have in the Topics a set of rules or laws called *ἀπὸ τοῦ μᾶλλον καὶ ἧττον καὶ ὁμοίως* which appear at least six times in that work. [45] Their interpretation offers some difficulty; alternatively to that followed below one might understand them as laws of logic of probability. The fundamental scheme is the following: "if A belongs more (equally, less) to x than to y, and it belongs (does not belong) to x, then it belongs (does not belong) to y". All laws stated here are substitutions of one of the following laws of the logic of propositions:

$$p \supset q . \sim q . \supset \sim p \qquad p \equiv q . \sim p . \supset . \sim q$$
$$p \supset q . p . \supset . q \qquad p \equiv q . q . \supset . p$$
$$p \equiv q . p . \supset . q \qquad p \equiv q . \sim q . \supset . \sim p$$

but they are not stated with such generality: for each variable a function of the type ARx is substituted, where A is a property, R the relation of inherence and x an individual. The important point here is that Aristotle seems to distinguish two kinds of

[42] *An. Pr. A 36, 48 b 11—14* — [43] *ib. 16—18.* — [44] *ib. 22—24.* The importance of these laws was pointed to by H. Scholz, *Geschichte.* — [45] *Top. B 10, 114 b 38ff.; Γ 6, 119 b 17ff.; 5, 127 b 18ff.; E 7, 136 b 34ff.; 137 a 8ff.; E 8, 137 b 14ff.; Z 7, 145 b 34ff.; H 1, 152 b 6ff.; 3, 154 a 4ff.*; cp. *Rhet. B 23, 1397 b 12ff.* The attention has been drawn to these formulae by Fr. Solmsen, *Entwicklung.*

inherence, a "stronger" and a "weaker". This might be interpreted in terms of probability; it seems, however, that he had in mind really two different relations. It is because of that distinction that these two laws, in spite of the fact that they are not, strictly speaking, laws of the logic of relations, may be quoted here, as Aristotle had in mind something like the inclusion of one relation in the other.

There are 18 laws of this kind, 3 formed of each of the above formulae. We give here only the three of the first group:

11.56. $AQx \supset ARy . \sim ARy . \supset . \sim AQx$
11.57. $AQx \supset BRx . \sim BRx . \supset . \sim AQx$
11.58. $AQx \supset BRy . \sim BRy . \supset . \sim AQx.$

11 F. Logic of propositions

We put at the end a few laws and rules of the logic of propositions, since Aristotle discovered and stated those most abstract theorems at the end of his evolution, when examining the structure of his axiomatic. They are four in number:

* 11.61. $p \supset q . \supset . \sim q \supset \sim p$ [46]

It is explicitly said the variables used ("*A*" and "*B*") refer to *sentences* [47] and in the same context truth is applied to them. [48]

11.62. $p \supset q . T\ulcorner p \urcorner . \supset . T\ulcorner q \urcorner .$ [49]

In the same context we find another important statement: "a true conclusion may be drawn from false premisses". [50] This is, however, neither $\ulcorner \sim p . \supset . p \supset q \urcorner$ nor $\ulcorner F \ulcorner p \urcorner . \supset . p \supset q \urcorner$.

11.63. $p_1 p_2 \ldots p_n . \supset . r : \supset : F\ulcorner r \urcorner . \supset . F\ulcorner p_1 \urcorner \vee F\ulcorner p_2 \urcorner \vee \ldots \vee F\ulcorner p_n \urcorner .$ [51]

This is an analogon of a generalized form of 8.21—22; it was

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[46] *An. Pr. B 2, 53 b 12ff.* — [47] *ib. 23f.* — [48] *ib. 20*, cp. *22.* — [49] *An. Pr. B 2, 53 b 13f.*; cp. *7f.* and *An. Post. A 6, 75 a 2—4.* — [50] *An. Pr. B 2, 53 b 7—9.* — [51] *An. Pr. B 4, 57 a 36f.*

meant as a rule of contraposition to be used in the reductio ad impossibile. But Aristotle stated it here for an indeterminate number of premisses.

In the modal logic we also have:

11. 64. $p \supset q . \supset . \Diamond p \supset \Diamond q$ [52]

which is also clearly stated as a law of logic of propositions. [53]

Further research would probably discover more non-analytical laws in the Organon, especially in the Topics.

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[52] *An. Pr. A 15, 34 a 5—7.* — [53] *ib. 17—19.* Another formula: $\ulcorner F \ulcorner p \urcorner . \sim I p . p \supset q . \supset . F \ulcorner q \urcorner \sim I \ulcorner q \urcorner$. (ib. *27—29*) seems to be invalid; cp. Becker *50ff.*

IV. THE OLD PERIPATETICIANS

12. Theophrastus and Eudemus

Theophrastus, with whom Eudemus is sometimes associated, was the most original logical thinker among Aristotle's pupils. The main lines of his system may still be reconstructed. We shall resume here, after some general introductory remarks (A), his most important theories, namely those concerning categorical syllogistics (B), his modal system (C), and the "hypothetical" syllogisms (D).

12 A. Introductory remarks

Theophrastus of Eresos († 288/7 or 287/6 B.C.), the chief pupil of Aristotle and first head of the peripatetic school after Aristotle's death, is reported to have written 20 logical works, [1] but only about 70 fragments of them are preserved by later writers, some of whom are not very reliable. It is possible, however, to get a general idea of what his logic must have been, and to recover some interesting doctrines which may safely be attributed to him. With Theophrastus, Eudemus of Rhodos is associated, but while there are many references to Theophrastus in our sources, we find only one and not a very important one refering to Eudemus, without the mention of Theophrastus.

In the light of the preserved fragments, we see that the work of Theophrastus consisted mainly in the development of the doctrines of Aristotle in the manner of Aristotle's own late writings. By doing so, Theophrastus contributed considerably to the formation of what was later called "classic logic" and perhaps also opened the path to the Stoic-Megaric Logic. At the same time, however, it must be stressed that his teaching contains several un-Aristotelian elements, especially in modal logic. He seems to have been, after Aristotle and some to the Stoic-Megaric school, the most important formal logician of Antiquity.

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[1] *DL 5, 42ff.*; cp. Bocheński, *Theophraste 26f.*

12 B. Doctrines concerning assertoric syllogistic

Only one fragment refers to the Theophrastian semiotics: it says that he distinguished a two-fold relation binding words with things and hearers respectively, i.e. he recognised the pragmatic dimension of the symbols.[2] He explicitly identified the indeterminate and particular sentences[3] and seems to have thoroughly examined sentences with negated predicates.[4] We learn also that he criticized Aristotle's proofs of the principle of contradiction.[5]

More important are the following doctrines. Theophrastus is reported to have stated the (false) principle:

12. 1. $$Ax \supset Bx . \equiv . (x)Ax \supset (x)Bx$$ [6]

and seems to have examined similar formulas which he called κατὰ πρόσληψιν. When examining the sentence "Phanias possesses science" he said that "science" must also be quantified,[7] thus introducing the beginning of a double quantification (the formula is of the type $R(a, x)$). In syllogistics proper he justified 9. 41 in the following way: if SeP, then P is separated (ἀπέζευκται) from S; thus S is also separated from P; and therefore PeS;[8] he seems, consequently, to have used something like a spatial diagram. He also explicitly stated five modes of the "indirect first-figure", namely 9. 67, 69, 68, 65, 66 (in this order),[9] and perhaps also 9. 72.[10]

12 C. Modal logic

This is the part of his theories which we know relatively best, probably because it struck the later authors as being the most original. In fact, we may safely attribute to Theophrastus two major changes in the Aristotelian logic: (1) the substitution of "$\Diamond$" for "E" in the syllogisms (while retaining the word ἐνδέχεται),

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[2] *Amm. De Int. 65, 31—66.* — [3] *ib. 90, 18ff.* — [4] *Alex. An. Pr. 396, 35—397, 4; Amm. De Int. 161, 5—11.* — [5] *Alex. Met. 273, 18ff.* — [6] *Alex. An. Pr. 378, 12ff.* — [7] *Waitz I, 40.* — [8] *Alex. An. Pr. 31, 4ff.; 34, 13ff.; Philop. An. Pr. 48, 11ff.* — [9] *Alex. An. Pr. 69, 26ff.*; cp. *109, 29ff.; Apul. De Int. 12, 193, 7—9; Boeth. De Syll. Cat. 815 b; Philop. An. Pr. 79, 10ff.* — [10] *Apul. De Int. 11, 189, 19ff.*

(2) the affirmation of the principle *peiorem semper sequitur conclusio partem* (τῷ ἐλάττονι καὶ χείρονι τῶν κειμένων ἐξομοιοῦσθαι) [11] with the result that all syllogisms of the 2nd and 3rd group have a Y-conclusion, all of the groups 4—8 a $\Diamond$-conclusion. In fact, the following laws of conversion and negation are preserved:

* 12. 11.	$N(SeP) \supset N(PeS)$ [12]
* 12. 12.	$N(SiP) \supset N(PiS)$ [13]
* 12. 13.	$\Diamond (SeP) \supset \Diamond (PeS)$ [14]
* 12. 14.	$\sim N(SaP) \supset \Diamond (SoP)$ [15]
* 12. 15.	$\sim N(SoP) \supset \Diamond (SaP)$ [16]
* 12. 16.	$\Diamond (SiP) \supset \sim N(SeP)$ [17]
* 12. 17.	$\sim \Diamond (SeP) \supset N(SiP)$ [18]

At the same time Theophrastus considered the Aristotelian laws 10. 37—39 as invalid [19]; this shows that he did not clearly distinguish between the two meanings of ἐνδέχεται. 12. 11 and 12. 13 were justified in the same manner as 9. 41.

Among the syllogistic modes we still have:

* 12. 21.	$N(PaM) . N(SoM) . \supset . N(SoP)$ [18]
* 12. 22.	$N(MoP) . N(MaS) . \supset . N(SoP)$ [18]
* 12. 23.	$N(MaP) . SaM . \supset . SaP$ [22]
* 12. 24.	$\Diamond (MoP) . MaS . \supset . \Diamond (SoP)$ [23]
* 12. 25.	$\Diamond (MaP) . N(SiM) . \supset . \Diamond (SiP)$ [24]
* 12. 26.	$N(MaP) . \Diamond (SaM) . \supset . \Diamond (SaP)$ [25]
* 12. 27.	$N(MeP) . \Diamond (SaM) . \supset . \Diamond (SeP)$ [26]
* 12. 28.	$N(PaM) . \Diamond (SoM) . \supset . \Diamond (SoP)$. [27]

A. Becker suggested that Theophrastus must have rejected the Aristotelian structure of modal sentences (10. 21 ff.) and have

[11] *Alex. An. Pr. 124, 8ff.* — [12] *Philop. An. Pr. 205, 13ff.* — [13] *Alex. An. Pr. 223, 4ff.* — [14] *Alex. An. Pr. 41, 21ff.; 220, 9ff.* — [15] *Alex. An. Pr. 126, 29ff.* — [16] *Alex. An. Pr. 123, 18ff.* — [17] *Philop. An. Pr. 205, 13ff.* — [18] *Alex. An. Pr. 223, 4ff.* — [19] *Alex. An. Pr. 123, 18ff.* — [22] *Alex. An. Pr. 132, 23ff.; 248, 3ff.* — [23] *Alex. An. Pr. 248, 19ff.* — [24] *Philop. An. Pr. 205, 13ff.* — [25] *Alex. An. Pr. 123, 18ff.* — [26] *Philop. An. Pr. 205, 13ff.* — [27] *Alex. An. Pr. 126, 29ff.*

conceived the modal functor as determining the copula. In fact this would explain the rule of the *peiorem* and several other details of his system. But since this rejection is nowhere reported, and Aristotle himself did not seem to have been perfectly clear about it, we must suppose that Theophrastus was led rather by a different intuition of the structure of modal sentences than by a explicitly formulated doctrine. The very fact that he so profoundly changed the Aristotelian system shows that the situation must not have been very clear for Aristotle himself.

12 D. Hypothetical syllogisms

It seems that the elaboration of a theory of alternative (*διαιρετικαί*) and conditional (*ὑποθετικαὶ κατὰ συνέχειαν*) [28] sentences may be attributed to Theophrastus and Eudemus. Both, especially Eudemus, are reported to have studied the "hypothetical syllogisms" extensively; yet among the various classes of such syllogisms which were known to Alexander, only one, the class of syllogisms called "analogical" (*κατ' ἀναλογίαν*) or "totally hypothetical" or, again, "hypothetical through all three" (*δι' ὅλου, διὰ τριῶν ὑποθετικαί*) [29] may safely be attributed to them. They are evidently developed out of 11. 33, yet, contrary to Aristotle, variables are used here. The first of these syllogisms is "if A, then B; if B, then C; thus, if A, then C". Thus — at least in the form we have them — they are not laws, but rules of inference. There is a difficulty in understanding what the variables stand for; but if we consider the corresponding text of Aristotle (11. 33) and the constant later tradition, it seems that they must be interpreted as predicate-variables, not as propositional variables. Thus we get five rules analogous to the following five laws:

* 12. 31. $Ax \supset Bx . Bx \supset Cx . \supset . Ax \supset Cx$ [30]

* 12. 32. $Ax \supset Bx . Bx \supset Cx . \supset . \sim Cx \supset \sim Ax$ [31]

* 12. 33. $Ax \supset Bx . \sim Ax \supset Cx . \supset . \sim Bx \supset Cx$ [32]

* 12. 34. $Ax \supset Bx . \sim Ax \supset Cx . \supset . \sim Cx \supset Bx.$ [33]

* 12. 25. $Ax \supset Cx . Bx \supset \sim Cx . \supset . Ax \supset \sim Bx.$ [34]

[28] This is the peripatetic terminology; cp. *Gal. Inst. 8, 6ff.*; *32, 11ff.*; Bocheński *108.* — [29] *Alex. An. Pr. 326, 20ff.* — [30] *ib. 22f.* — [31] *ib. 326, 37—327, 2.* — [32] *ib. 327, 17ff.* — [33] *ib.* — [34] *ib. 327, 8f.*

These laws were classed by Theophrastus into figures, namely 12. 31—32 belonged to the first, 12. 33—34 to the second, and 12. 35 to the third.

It is also not improbable that Theophrastus elaborated — working out ideas embodied in theorems like 11. 1 — three more such syllogisms which are attributed to him by a rather unreliable source: [35]

12. 36. $(X). MaX \supset XaP : MaS : \supset : SaP$

12. 37. $(X). MaX \supset PaX : MaS : \supset : PaS$

12. 38. $(X). XaM \supset XaP : SaM : \supset : SaP.$

It is not impossible, finally, that he elaborated the Aristotelian 11. 56 ff. with variables. [36]

[35] *Amm. An. Pr. XIII*; cp. Bocheński *117ff.* — [36] Philippson; cp. Bocheński *25, 119f.*

V. THE STOIC-MEGARIC SCHOOL

The development of formal logic in Antiquity reached its peak in the works of the thinkers belonging to the Megaric and Stoic Schools. Unfortunately, none of those works are preserved and our information concerning them supplied by later sources is desperately scarce. It is sufficient, however, to show that among both Megaricians and Stoics there were very great logicians and that the general level of the formal rigour obtained by those schools was remarkable — indeed, superior in some respects to that of our own today. Among the discoveries which may safely be attributed to them, are the following: invention and statement in form of an axiomatic system (which seems to have been both consistent and complete) of a logic of propositions; invention of truth-tables and thorough discussions of the meaning of implication; subtle semiotical doctrines, including a sharp distinction between the logical laws and the metalogical rules of inference, and a clear distinction between intension and extension.

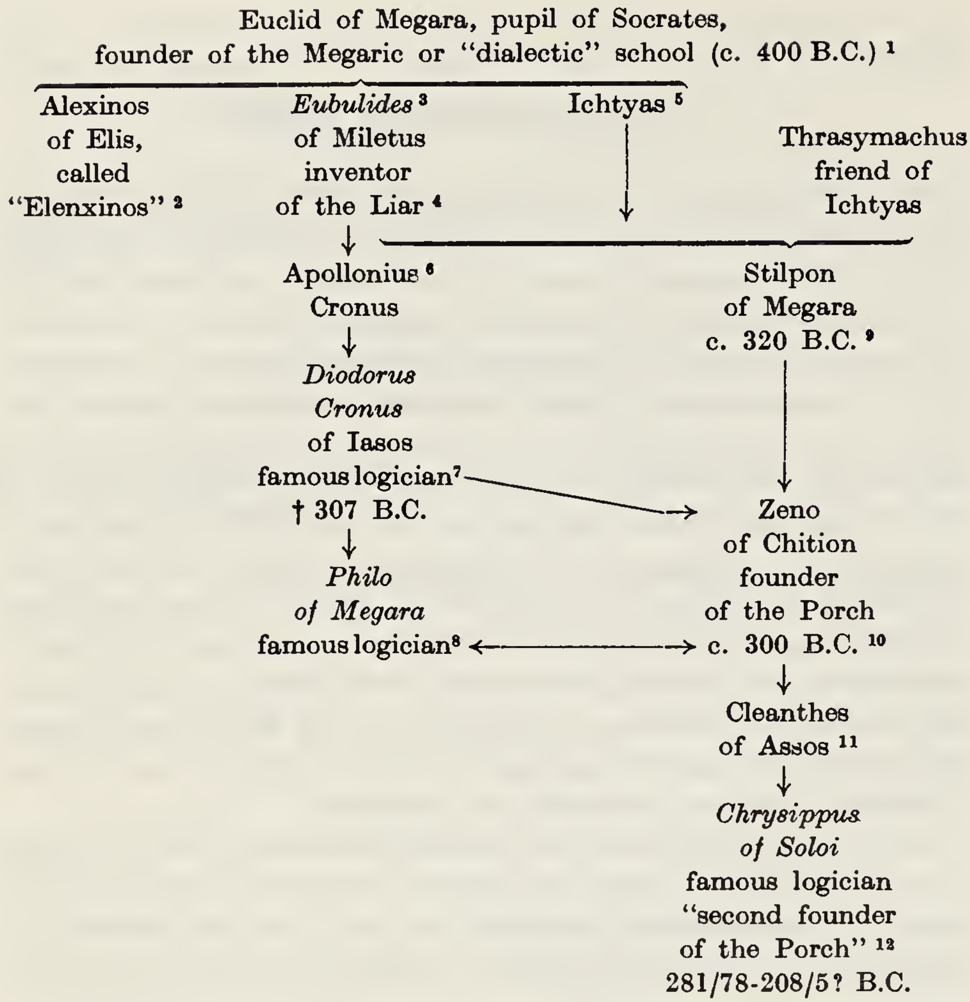
We shall expound here, after a historical survey (13), their logic in four chapters, dealing respectively with semiotics (14), the theory of propositional functors (15) the rules of inference or syllogisms (16) and the paradoxes, including the famous Liar (17).

13. Historical Survey

We shall collect here some data concerning the external history of the Stoic-Megaric School, naming and showing the interdependence of these thinkers (A), giving the sources for their teaching (B); then we shall attempt a general characterisation of their logical doctrines (C), and, finally, advance a hypothesis as to the origin of those doctrines (D).

13 A. The thinkers. Megaricians and Stoics

The genealogy of the schools, as far as logic is concerned, may be represented by the following scheme which is based on Diogenes:



This is practically all we know about the logicians of both schools, for after Chrysippus the Megaric School seems to have been extinct, while the Porch, if it was very flourishing, still did not have a single logician whose fame has reached us. We may note that the Megaricians seem to have been in some respects superior to the Stoics, for against three of their celebrated logicians — Eubulides,[13] Diodorus[14] and Philo[15] — we know of only one

[1] *DL 2, 106ff.* — [2] *DL 2, 109.* — [3] *DL ib.* — [4] *DL 2, 108.* — [5] *DL 2, 112.* — [6] *DL 2, 111.* — [7] *ib.*; *Epict. 2, 19, 1*; *Cic. De Fato 7, 13.* — [8] cp. *DL 7, 16.* — [9] *DL 2, 113ff.* — [10] *DL 7, 1ff.* — [11] *DL 7, 168ff.* — [12] *DL 7, 179ff.* — [13] cp. ch. 17. — [14] cp. ch. 14 E (1), 15 C (2). — [15] cp. ch. 14 E (2), 15 C (1).

great Stoic thinker: Chrysippus. Moreover, while important doctrines can be ascribed to the former, nothing of that kind may be attributed with any certainty to Chrysippus as inventor. Finally, there is no doubt that Zeno himself learned logic from Diodorus [16] and that the whole movement depends on the "dialectic" school of Megara. However, as the Megaric school disappeared and as Stoicians cultivated logic during a long time, the whole doctrine came to be called "Stoic logic". It seems more correct to call it "Stoic-Megaric". This does not mean that there were two different schools; it is more probable that in spite of some differences (which existed also inside of the Megaric School) the bulk of the doctrines was common to both groups and that the opposition is rather chronological than systematic.

Chrysippus merits a special mention. All his writings, reported to have been more than 705 in number [17], are lost, and we know very little of his teaching. Yet, we know that he was recognized throughout the Antiquity as a powerful logician, to the point that it was said "if there were no Chrysippus, there would have been no Stoa" [18] and "if gods have logic, this must be Chrysyppean". [19] It is, indeed, not impossible that with him ancient formal logic reached its highest level of insight and rigour.

13 B. Sources

As already said, all works of the Megaricians and Stoics are lost, and we are compelled to use reports about their doctrines by later writers. Among those reports two have an outstanding importance. The first is contained in the 7th book of the "Lives of Philosophers" by Diogenes Laertius, who lived apparently in the third century A.D. This report is drawn, however, from Diocles Magnus (1st century B.C.) and contains much information about Stoic logic. More important still are the works of Sextus Empiricus, a sceptical physician who seems to have flourished about 150 A.D. He had a keen interest for, and a relatively good understanding of, the Stoic-Megaric logic: the 8th book of his work "Against

[16] *DL 7, 25.* — [17] *DL 7, 180.* — [18] *DL 7, 180.* — [19] *DL 7, 183.*

Mathematicians" (i.e. against people who sustain any affirmation) is the most detailed report we have, and, in spite of several difficulties it offers, it is by far the best. There are also many secondary sources: thus we have some mentions in Cicero (1st cent. B.C.), Seneca († 65 A.D.) Alexander Aphrodisias and Galenus (2nd cent. A.D.). The two last named, if their understanding of the Stoic-Megaric doctrines is sometimes dubious, have still a considerable importance, especially Galenus, whose booklet is the only preserved Greek textbook of logic of the post-Aristotelian ancient period. Later writers and commentators supply several references, but their understanding of the subject is generally bad.

As stressed, all these sources together supply only a very fragmentary view of the Stoic-Megaric logic. Thus the famous "master-argument" of Diodorus is lost, and out of the "innumerable" inference rules elaborated by the Stoics, no more than a dozen are left. Moreover, it must be pointed out that most of the reports we have are by men by no means sympathetic to these thinkers, and in some cases (as Sextus') violently opposed to them.

13 C. General characteristics

Yet, along with many details, we are able to see that the logic evolved by this school had a few important features by which it sharply differed from the Aristotelian logic. The most important among them, ignored by Prantl, and by practically everybody until 1923, were discovered by J. Łukasiewicz. They may be resumed as follows: (1) While the Aristotelian logic corresponds in its main part (syllogistics) to what is called today "logic of classes" (or "of predicates"), all extant theorems of the Stoic-Megaric School belong to the *logic of propositions.* (2) While Aristotle stated most of his theorems in the form of conditional propositions (or functions) expressed in the object-language, the Stoics formulated *rules of inference* and used metalogical language to do so. (3) While with Aristotle the ontological status of the formulae is not adequately determined (we do not know if they are sequences of words, or mental or objective structures) Stoics elaborated a refined semiotic theory and stated their logical theorems in such a way that they would always mean something belonging to the realm of meanings (*λεκτά*). (4) Finally, they introduced, probably for reasons connected with their competition with the peripateticians, a completely new terminology: where Aristotle used "*πρότασις*" they said "*λῆμμα*"; where he had "*συμπέρασμα*" they put "*ἐπιφορά*" and so on.

13 D. The origin

This completely new terminology, combined with the new subject and technique of exposition of logic, is the cause of the feeling that we have here something radically different from the peripatetic doctrine. In Antiquity this difference was even felt as an opposition. We know today that there is no such opposition and that the Stoic-Megaric school simply developed aspects and parts of logic which Aristotle did not study, while those elaborated by him seem to have been completely omitted by them. Thus — and this is a rather curious fact — we *never* hear any thing about their logic of predicates or classes; it seems that this did not exist.

Moreover, it is not difficult to see that the Stoic-Megaric doctrine was — at least in part — developed out of the Aristotelian teaching. We have seen that Aristotle already stated a few theorems belonging to the logic of propositions; we know that he recommended the study of them. Now most of the still-preserved Stoic theorems are such that either their analoga are to be found in Aristotle or that they were evolved most naturally out of his practice: Thus, out of the two preserved *θέματα* one is a re-statement of an Aristotelian rule, while the other was constantly used by Aristotle in his "direct" reduction of syllogisms. It seems also that Theophrastus might have built the bridge between Aristotle and the Stoics, in so far as he introduced everywhere variables and was busy with syllogisms based on hypotheses.

But Aristotelian logic was probably only one of the sources of the Stoic-Megaric doctrines. Much of it must have been directly developed out of the inference schemes used by the pre-Aristotelian. We know that the first Megaricians were very keen dialecticians;

it is not improbable that they explicitly stated rules of the logic of propositions which had been for a long time in common and fairly conscious use. Thus we may advance the hypothesis that the two main sources of the Stoic-Megaric logic was the late-Aristotelian teaching and the λόγοι in common use by the dialecticians of the time following Socrates.

14. Notion of Logic: Semiotics: Modalities: Categories

We shall expound here different doctrines which may be considered as introductory to the Stoic-Megaric formal logic proper: the Stoic notion of Logic (A), Semantics (B), the classification of the objective meanings (*λεκτά*) (C), the uses of "truth" (D), the various definitions of modalities (E), and, finally, the Stoic theory of categories (F). Among them, the doctrines concerning meaning and modalities are of a remarkable subtelty, while others have considerable (although at times only historical) importance.

14 A. Notion of Logic

We do not know how the Megaricians defined Logic, while on the contrary, there are abundant sources for the corresponding Stoic doctrine. Logic was to them not only an instrument (*ὄργανον*) of philosophy, [1] but also one of its main parts (*μέρος*), which were: Logic (*λογικὸν μέρος*), physics, and ethics. [2] Logic was defined as "the science of (entities) which are true, false of neither" [3] which means: of propositions and their parts. [4] Its subject were the arguments (*λόγοι*) and its aim the knowledge of the demonstrative methods (*ἀποδεικτικοὶ μέθοδοι*). [5] Logic was again divided into rhetorics and dialectics (*διαλεκτική*); the latter was defined as the science of correct discourse (*τοῦ ὀρθῶς διαλέγεσθαι*) in arguments concerning questions and answers. [6] But Stoic dialectics covered a wider field than our formal logic — it included also theory of knowledge, with an ample criteriology and much psychology of knowledge.

14 B. Semantics

The Stoics developed a highly complex and refined semiotics. They distinguished three factors in the semantic situation: the symbol (*τὸ σημαῖνον*), a material sound; the significate or meaning (*τὸ σημαινόμενον*), and the external thing itself (*τὸ πρᾶγμα*,

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[1] *Amm. An. Pr. 8, 20ff.* — [2] *DL 7, 39*; *Amm. An. Pr. 9, 1ff.*; *Aetii pl. I, Proem 2*, cp. *SVF 35.* — [3] *DL 7, 42.* — [4] This correct interpretation is due to Dr Mates. — [5] *Amm. An. Pr. 9, 1ff.* — [6] *DL 7, 41.*

τυγχάνον).[7] The significate, which was also called "that which is said" (*τὸ λεκτόν*), was considered as incorporeal in opposition to the thing and the symbol, which were both bodies.[8] They distinguished speech as a physical phenomenon (*λέγειν*) from speech as a vehicle of meaning (*ἀπαγορεύειν*)[9]. Signs (*σημεῖα*) were also divided into commemorative and indicative (*ἐνδεικτικά*)[10]; the latter were considered as true antecedents of true conditional propositions (*προκατηγούμενον*)[11]; e.g. "she has milk" was considered as the true antecedent of the true proposition meant by "if she has milk, she has conceived".[12]

The *λεκτά* were said to be incorporeal[13]; since the Stoics did not admit other objects than bodies, this gave occasion for "unending" discussions about the existence of the *λεκτά*.[14] The definition of a *λεκτόν* was: "what consists in conformity with a rational presentation (*φαντασία λογική*)[15] i.e. "an object as conceived".[16] It may be said that the *λεκτόν* corresponded to the intension or connotation of the words.

Chrysippus held that all words are ambiguous[17]; he wrote a number of books on the subject[18] and distinguished seven kinds of amphiboly[19]; the Stoic name for amphiboly was *πολλὰ ἅμα ἔχοντα ὀνόματα*[20]. But Diodorus is reported to have held that no word is ambiguous.[21] The problem must have been amply discussed.

14 C. Classification of the *λεκτά*

Here is a scheme representing the Stoic classification of the various kinds of *λεκτά*:

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[7] *AM 8, 11 = SVF 166.* — [8] *ib.* cp. *AM 7, 38 = SVF 132, 22.* — [9] *Plut. De Stoic. 11, 1037 d = SVF 171.* — [10] *AM, 8 143.* — [11] *HP B 104.* — [12] *HP B 106.* — [13] *AM 8, 11 = SVF 166; AM 7, 38 = SVF 132* etc. — [14] *AM 8, 262,* cp. *258.* — [15] *AM 8, 70 = SVF 187; DL 7, 63 = SVF 181.* — [16] *AM 8, 80.* — [17] *Gellius 11, 12 = SVF 152.* — [18] cp. *SVF 14.* — [19] *Gal. De Soph. 4, vol. 14, 595 = SVF 153.* — [20] *Simpl. Cat. 36,9ff. = SVF 150.* — [21] *Gellius 11, 12 = SVF 152.*

what is said (meant): *λεκτόν*

- deficient *ἐλλιπές* [22]
  - what is meant by a noun (*προσηγορία*): *πτῶσις*? [23]
    - common: *κοινὴ ποιότης* [25]
    - individual: *ἰδία ποιότης* [26]
  - what is meant by a verb (*ῥῆμα*): *κατηγόρημα* [23]
- complete *αὐτοτελές* [22]
  - proposition: *ἀξίωμα* [24]
    - atomic: *ἁπλοῦν* [27]
      - various divisions according to the negation [29]
      - Definite: *ὡρισμένον* e.g.: "this man walks"
      - intermediate: *μέσον* e.g.: "Socrates walks"
      - indefinite: *ἀόριστον* e.g.: "somebody walks". [30]
    - molecular (subdivided according to the functor) [28]
  - other: *πύσμα* etc. [24]

It will be seen from the above that the Stoic classification corresponds closely to that of Aristotle [31]. There is one capital difference between both, however: while in Aristotle's classification we have to do with (meaningful) words, the Stoics were emphatic about the point that they were dealing here, as in the whole of their logic, with meanings (*λεκτά*). We did not mention in our scheme the Stoic division of molecular sentences which will be explained in the next chapter and is probably one of their most important merits in logic.

14 D. Truth

The Stoics asserted truth of different entitities, and their doctrine of truth — at least as it appears in our fragments — is a rather confused one. (1) First of all they are reported to have held that truth is "in" or "about" propositions. [32] (2) A propositional function is said to be true (*ἀληθές*) for some or all of its values. [33]

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[22] *DL 7, 63 = SVF 181*; *Philo De agr. 139 = SVF 182.* — [23] *DL 7, 57f. = SVF 183.* — [24] *AM 8, 70 = SVF 187*; *DL 7, 66 = SVF 186.* — [25] *DL 7, 58.* — [26] *ib.* — [27] *AM 8, 93 = SVF 205*; *DL 7, 68 = SVF 203.* — [28] cp. chap. *15 Cff.* — [29] cp. chap. *15 B.* — [30] *AM 8, 96—100 = SVF 205.* — [31] cp. chap. *5 C.* — The above scheme is essentially due to Dr. Mates. — [32] *AM 8; 11* and *70*; *DL 7, 66*; *Simp. Cat. 406, 22.* — [33] *Boeth. 234 = SVF I, 489.*

(3) Third, presentations (φαντασίαι) were said to be true.[34] (4) Finally, an argument (λόγος) was said to be true if and only if it was valid and had true premisses.[35] Thus it seems that the basic notion of truth was to them that of propositions.

We find also in Stoic fragments a curious distinction between truth (ἀλήθεια) and the true (ἀληθές)[36]. While truth was (a), a body — for it was knowledge, i.e. a part of the soul which was said to be a body — (b), composed of many truths, and (c), found only in good men — the true being a λεκτόν, was incorporeal, simple, and to be found also in bad men. It is also worth noting — contrary to what Rüstow thought — they had a distinction between falsehood and lie.

14 E. Modalities

It seems that the problems of modalities were most discussed by the Megaricians, but the Stoics also took part in the research and advanced their own point of view. Three different theories are preserved; all of them are connected with discussions about the truth of the conditional proposition, i.e. with the definition of implication.

(1) *Diodorus Cronus* defined the modalities with the use of a time variable. His definitions[37] may be stated in the following terms, according to Dr Mates:[38]

14. 11. $\ulcorner p(t) \urcorner$ is possible at $t_n . =_{Df} . p(t_n) \vee (\exists t) . t_n < t . p(t)$

14. 12. $\ulcorner p(t) \urcorner$ is impossible at $t_n . =_{Df} : \sim p(t_n) . (t) : t_n < t . \supset . \sim p(t)$

14. 13. $\ulcorner p(t) \urcorner$ is necessary at $t_n . =_{Df} : p(t_n) . (t) : t_n < t . \supset . p(t)$

14. 14. $\ulcorner p(t) \urcorner$ is non-necessary at $t_n . =_{Df} : \sim p(t_n) \vee (\exists t) . t_n < t . \sim p(t)$.

It will be seen that the four modal functions thus defined form a logical square as in the Aristotelian *De Int.*[39]. Diodorus attempted to prove the correctness of 14. 11 by his "master-argument" (ὁ κυριεύων) which is lost; we know only from Epictetus[40] that it contained the following assertion: "the propositions, (1) every true

[34] *AM 7, 244.* As taken in one of those three senses ἀληθές was interchangeable with ὑγιές (*AM 8, 111ff., 125, 245ff.*) — [35] *HP B 138*; *DL 7, 79*; *AM 8, 411 f.* — [36] *HP B 81*; *AM 7, 38ff.* — [37] *Boeth. De Int. 234.* — [38] Mates, *Implication, 236ff.*; the symbolism has been changed. — [39] cp. *10 C.* — [40] *Epict. Diss. 2, 19, 1.*

proposition about the past is necessary, (2) an impossible proposition does not follow from a possible proposition, (3) there is something which is possible and yet neither is nor will be true — cannot all be true (there is a *κοινὴ μάχη . . . πρὸς ἄλληλα*)". Diodorus himself preferred the first two and rejected the third — but we do not know why.

(2) According to *Philo* the Megarician [41] a proposition was possible if and only if it was susceptible of truth by its internal nature. He defined the necessary as follows: "that, which being true, is in its very nature not susceptible of falsehood".

(3) We know very little about the view of *Chrysippus*; he perhaps agreed with Philo [42]. Dr Mates suggested, however, that a different view, stated in the "Lives" of Diogenes (7. 75) might be attributed to him. According to that view a possible proposition is one which admits of being true when external circumstances do not prevent its being true; a necessary proposition is one which being true does not admit of being false, or is prevented from it by external circumstances. This looks very much like Philo's view, but was, nevertheless considered as a different one.

14 F. Categories

We shall finally just mention the Stoic doctrines of categories, which cannot be fully stated before a thorough study of the very fragmentary tradition is undertaken. The categories (*τὰ γενικώτατα*) do not seem to belong to Stoic logic, but rather to their physics [43]. There is, according to them, a supreme genus, the something (*τὸ τί*) divided into four main categories: the subject (*τὸ ὑποκείμενον*), quality (*τὸ ποιόν*), state (*τό πως ἔχον*), and relation (*τὸ πρὸς τί πως ἔχον*) [44]. These four categories are so related to one another, that the preceding category is contained and determined by the next succeeding it. [45] It seems also that the Stoics attempted something which would be a combination of the two classifications of Aristotle; but what their doctrine really was, we do not know.

[41] *Boeth. De Int.*[2] *234*; *Simpl. Cat. 195f.*; *Alex. An. Pr. 184, 6f.* — [42] *Cicero De Fato 12.* — [43] V. Arnim seems to be right in collecting the respective fragments in his chapter on physics (*SVF 2, 369—375*). — [44] *Plot. Enn. 6, 1, 25*; *Simpl. Cat. 66,32—67,8.* — [45] cp. Trendellenburg, *Gesch. 220ff.*; Zeller, *Stoics, 109ff.* where the sources are collected.

15. Propositional functors

One of the most remarkable achievements of the Stoic-Megaric School is the theory of molecular propositions developed by means of an exact analysis of the meaning of propositional functors. We shall expound here, after a general survey (A), their theory of negation (B), implication (C), disjunction (D) conjunction and other functors (E), and state a number of definitions of one functor in terms of others (F).

15 A. General survey.

The Stoic-Megaric logic was consciously built up on an analysis of logical properties of propositional functors (i.e., more exactly, of what is meant by propositional functors). These properties were studied by examining the problem of the truth of molecular propositions built up with such functors. There was much discussion about such problems [1]: Callimachus, the head of the Alexandrian library (260—240 B.C.) is reported to have said that "even the crows on the rooftops are cawing over which conditional is true" [2]. In the course of those discussions several quite correct truth tables were stated.

The molecular propositions were said to be compounded out of atomic ones by different connectives (*σύνδεσμος*), which were said to "announce" (*ἐπαγγέλλειν*) something about the parts of the molecular proposition in question. The main classes of molecular propositions examined were: the conditional (*συνημμένον*), the disjunctive (*διεζευγμένον*), and the copulative (*συμπεπλεγμένον*), but there were still others of lesser importance. Although these logicians did not consider negation together with the connectives, we shall briefly touch upon it here.

15 B. Negation

Negation was examined when opposite propositions (*ἀντικείμενα*) were treated. The Stoics distinguished: (1) the negation (*ἀποφατικὸν ἀξίωμα*), formed from a proposition by prefixing the "not" (*οὐχί*);

[1] *Cic. Acad. Pr. 2, 143.* — [2] *AM I, 309.*

they greatly insisted that this "not" must be placed at the beginning of the denied proposition; (2) the denial (*ἀρνητικόν*) was a proposition composed of a negative particle (as "no-one", *οὐδείς*) and a predicate, e.g. "no-one walks"; (3) the privative proposition (*στερητικόν*) was a proposition in which the subject was qualified by a (term-)negation, e.g. *ἀφιλάνθρωπός ἐστιν οὗτος* [3]. Two propositions were said to be contradictory when one was richer than the other by the negation. [4] We know of no attempt to construct a truth-table for negation; on the other hand, while discussing the double negation (*ὑπεραποφατικόν*), the Stoics seem to have stated the principle of double negation

15. 1 $$\sim\sim p \equiv p.$$ [5]

15 C. Implication

A conditional is, according to the Stoics, a molecular proposition composed of two propositions, namely of the "first" (also called "*ἡγούμενον*") and the "second" (also "*λῆγον*") — connected by the "if" ("*εἰ*" or "*εἴπερ*"), which "announces" that the second follows from the first. [6] This was the common doctrine; but there were widely divergent opinions as to the meaning of "follows" (*ἀκολουθεῖν*) used in that generic definition and, consequently, there arose different definitions of implication. [7] Thanks to a text of Sextus [8] we still know four such definitions, namely those of Philo, of Diodorus, and of two others, which may be called (with Dr Mates) the "connective" and the "suggestion" view.

(1) According to *Philo* "a conditional is true if and only if it does not have a true antecedent and a false consequent". [9] We find in Sextus a complete truth-table stated in order to explain this definition [10]; this is exactly our truth-table of material implication, as Peirce has already remarked. [11]

(2) *Diodorus* defined a true conditional as "one which neither

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[3] *DL 7, 69.* — [4] *AM 8, 89; DL 7, 73.* — [5] *DL 7, 69.* — [6] *AM 8, 109; DL 7, 71.* — [7] *HP B 110ff.; AM 8, 112.* — [8] *HP B 110—112*; cp. *AM 8, 112—117.* — [9] *HP B 110; AM 8, 113f.* — [10] *AM 8, 245—247.* — [11] Peirce *II, 199; III, 279f.*

is nor ever was capable of having a true antecedent and a false consequent" [12]. This — in view of 14. 13 — may be stated, following Dr Mates who examined the problem [13], in the following terms (where "→" stands for the Diodorean implication):

15. 2. $$p \rightarrow q . =_{Df} . (t) . p(t) \supset q(t).$$

It will be seen that the Diodorean implication is somewhat stronger than Professor Lewis' strict implication.

(3) The *"connective" view* (that of *συνάρτησιν εἰσάγοντες*) is stated in the following terms: "a conditional holds whenever the denial of its consequent is incompatible with its antecedent" [14]. Incompatibility (*μάχη*) means here clearly impossibility, because otherwise this definition would mean the same as the Philonian.

Thus we have here:

15. 3. $$p \prec q . =_{Df} . \sim \Diamond (p \sim q)$$

which is Lewis' definition of strict implication. [15]

(4) The suggestion view (that of *ἐμφάσει κρίνοντες*) is thus described: "a conditional is true if its consequent is potentially included (*περιέχεται δυνάμει*) in its antecedent" [16]. Consequently, "if p then p" was said to be false by the partisans of that definition, as no proposition includes itself. [17] We have no further information about this view.

It may be further noted that Fr., Stakelum discovered in Galenus the use of "conditional" (*συνημμένον*) as meaning "bi-conditional" [18]; the "if" means there, consequently, "if and only if" (equivalence). Dr van den Driessche found a similar use in Boethius. [19]

15 D. Disjunction

The disjunctive proposition was defined as one formed by the conjunction "or" ("*ἤτοι*") [20]; this had, however, several meanings.

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[12] *HP B 110; AM 115ff.* — [13] Mates, *Implication.* — [14] *HP B 111.* — [15] Lewis-Langford *244.* — [16] *HP B 112.* — [17] *ib.* — [18] Stakelum *48—53, 73f.* (*Gal. Inst. 9*). — [19] van den Driessche *294ff.* — [20] *DL 7, 72.*

The Stoics knew two fundamental kinds of disjunction: the exclusive (*διεζευγμένον*) and a weaker form called "*παραδιεζευγμένον*". The first was then currently defined by means of the following: "the disjunctive proposition is true if one of its parts is false" [21] or "if it has (just)one true part"[22]. This is rather vague, but another text says "if one part is true and other or others false or false and incompatible" [23], and out of the fourth and fifth undemonstrables (16. 24—25) which were fundamental in Stoic logic, we see that exclusive disjunction (matrix "*0110*") was meant. There are, however, other texts [24] indicating another opinion according to which a disjunctive proposition was true if its components could not be both true, i.e. that those logicians defined the disjunction by the matrix "*0111*" (Sheffer's functor). [24]

The weaker variety, *παραδιεζευγμένον*, seem to have been our logical sum (matrix "*1110*"), but the fragments preserved [25] are far from being clear.

Let us note further that Chrysippus asserted with great energy [26] the *tertium non datur* in which "or" has the exclusive meaning. [27] The form of his statement is:

15. 4. $(p).T\ulcorner p\urcorner \vee F\ulcorner p\urcorner.$

From a fragment of Apollonius we know that commutativity of disjunction was explicitly stated:

15. 5. $p \vee q.\supset.q \vee p.$ [28]

15 E. Conjunction and other functors

A conjunctive proposition was defined by the Stoics as one compounded by "and" (*καί*) [29]; it was true just if all its parts were true [30] i.e. the functor was defined by the truth-table "*1110*" as our logical product (only an indeterminate number of arguments was meant).

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[21] *DL 7, 72.* — [22] *AM 8, 282, Gell. 5, 11; 16, 8.* — [23] *HP B 191.* — [24] *Gal. Inst. 11f.; Amm. An. Pr. XIf.; Bekker Anecd. II, 485.* — [25] *Gal. Inst. 33, 10ff.* — [26] *Cic. De Fato 10, 20f.* cp. *16, 38* — [27] *DL 7, 65; Simpl. Cat. 406, 22ff; Cic. Tusc. 1, 7, 14; De Fato 10, 20.* — [28] *Bekker Anecd. II, 485.* — [29] *DL 7, 72; Gal. Inst. 10, 15ff.* — [30] *AM 8, 125; Gell. 16, 8.*

Along with the three principal functors described, many others were in use. First, Stoics had an inferential proposition (*παρασυνημμένον*) compounded by means of "since" ("*ἐπεί*"), which meant that the second follows from the first and that the first is true. [31] This offers some difficulty because if Philonian implication was meant, the inferential functor would not differ from the functor of conjunction. Second, there was a number of other functors which cannot be defined by truth-tables, namely the causal (*αἰτιῶδες*) compounded by means of the "because" (*διότι*) [32], and two "comparative" propositions: that which "declares the more" (*διασαφοῦν τὸ μᾶλλον*) and "the less" (*δ. τὸ ἧττον*). [33] The list is probably incomplete.

15 F. Definibility of the functors in terms of one another

Stoic logicians examined also the relations existing between their truth-functors, and we still have three equivalences (or perhaps, definitions) stated by them:

* 15. 6. $p \supset q . \equiv . \sim (p \sim q)$

This is explicitly attributed to Chrysippus by Cicero who finds the thesis ridiculous. [34]

* 15. 7. $p \vee q . \equiv . \sim p \equiv q$

is referred to in a rather difficult text by Galenus. [35] Fr. Stakelum, who made a thorough-going analysis of that text [36] has shown that "bi-conditional" is meant here by "conditional" (*συνημμένον*). Another text [37] seems to support the view that

15. 8. $p \vee q . \equiv . p \supset \sim q . \sim q \supset p$

was also a Stoic thesis.

There were probably more such definitions or equivalences. On the other hand we know of no attempt to build an axiomatic system of these formulae.

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[31] *DL 7, 71, 74; Amm. An. Pr. XIf.* — [32] *DL 7, 72.* — [33] *ib.* — [34] *Cic. De Fato 8, 15f.* — [35] *Gal. Inst. 9ff.* — [36] cp. Stakelum *48ff.; 73f.* — [37] *Bekker, Anecd. II, 489, 2ff.* Actually we have $p \vee q . \equiv . p \supset \sim q . q \supset \sim p$; cp. Mates.

16. Arguments and schemes of inference

The central part of the Stoic logic was constituted by the theory of arguments and of schemes of inference, of which these arguments are substitutions; like the other chapters of their logic, this was also highly technical and formal. We shall deal here with the definition and division of arguments (A), the schemes of inference and their axiomatization (B), the celebrated "undemonstrables" (C), and derived modes (D).

16 A. Definition and division

An argument (*λόγος*; however the same word also means a sentence) is, according to the Stoics, a system (*σύστημα*) composed of premisses (*λήμματα*) and conclusion (*ἐπιφορά*) [1]. It must be stressed that the premisses are certainly not sentences, but propositions; thus the argument must have been conceived as a system of *λεκτά*, not as a system of words. On the other hand, an argument itself was certainly not a proposition as was Aristotle's syllogism; in fact, we are told, that an argument is valid (*συνακτικός*) when a conditional proposition (*συνημμένον*) which has its premisses as antecedent and its conclusion as consequent is true. [3] This shows clearly that the argument was not identical with that proposition. The truth concerned in the above definition, as Sextus gives it, is Diodorean; Diodorus says that the corresponding proposition "never begins with truth and ends with falsehood". [4] But Sextus himself says that there was no agreement among Stoicians about that point [5] — and indeed, in face of the different definitions of the implication [6], it could not have been otherwise.

A valid argument was called "true" (*ἀληθής*) when its premisses were true; consequently a false argument was one which was either invalid or had false premisses. [7] Again, a true argument

[1] *DL 7, 45*; but *DL 7, 76* says that according to the followers of Crinis an argument is composed of (one or more) *λήμματα*, of a *πρόσληψις* (minor premisses) and of an *ἐπιφορά*; cp. e.g. *Amm. An. Pr. 68, 4—8*. — [3] *HP B 137*; *AM 8, 415, 426*. — [4] *AM 8, 416, 419*. — [5] *HP B 145*; *AM 8, 426f*. —[6] cp. ch. 15 *C*. — [7] *HP B 138, 187*; *AM 8, 421, 414*; *DL 7, 79*.

was called "demonstrative" ("*ἀποδεικτικός*") when it revealed a non-evident conclusion (*τὸ τὴν ἐπιφορὰν ἄδηλον οὖσαν ἐκκαλύπτεσθαι*)[8]. On the other hand, some of the valid arguments were called "undemonstrated" ("*ἀναπόδεικτοι*"). Those were either such which could not, or which need not be demonstrated[9]; the second class — which is the only relevant one — was composed of arguments in which it is immediately clear (*αὐτόθεν περιφανές*) that they are valid (*τὸ ὅτι συνάγουσιν*)[10]. There is one text of Sextus in which those undemonstrated are further divided into simple (*ἁπλοῖ*) and non-simple (*οὐχ ἁπλοῖ*); the latter, he says, need reduction (*ἀνάλυσιν*) into the first.[11] Thus the whole division, including that text, would be the following:

- arguments
  - valid
    - true — false
      - demonstrative
      - other
    - undemonstrated
      - simple
      - non-simple
    - others
  - invalid

But Diogenes opposes the undemonstrated to the "reducible to undemonstrated"[12] and the other texts of Sextus do not support his division, while if we admit that this division is correct, there is a great difficulty in understanding what "undemonstrated" may mean. Thus it seems preferable to consider the text of Sextus in question as a corrupted one.

Among lesser classes of arguments we may still note those which "conclude un-methodically" (*ἀμεθόδως περαίνοντες*) i.e. were not fully stated[13]; the one-premissed (*μονολήμματα*), such as "you are breathing; thus you are living", which were much discussed[14]; the so-called "duplicated" (*διφορούμενοι*) arguments of the form of e.g. $\ulcorner p \supset p.\ p.\supset p \urcorner$[15] and the unanalyzed (*ἀδιαφόρος περαίνοντες*) which were perhaps the same as the "unmethodical".

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[8] *AM 8, 422, 311f.* — [9] *AM 8, 223.* — [10] *ib.* — [11] *AM 8, 228.* — [12] *DL 7, 78.* — [13] *Alex. An. Pr. 345, 13ff.; Top. 6.* — [14] *AM 8, 443; HP B 167; Apul. 272; Alex. An. Pr. 17, 12—15; 21, 25ff.* — [15] *Alex. Top. 10; Cicero Ac. Pr. 2, 96.*

16 B. The logical status of arguments

We have already seen that the stoic arguments were neither sequences of words (14) nor conditional propositions; the latter is in direct opposition to the Aristotelian practice (8Al). We have seen also that the argument is, according to the Stoics, a "system" of propositions. In fact its verbal symbol has always the following form: $\ulcorner p$; but q; therefore (ἄρα) $r\urcorner$. Łukasiewicz, who was the first to point this out in modern times, stated that while Aristotelian syllogisms were substitutions of laws (tezy, Thesen), the stoic arguments must be considered as substitutions of rules of inference (reguly wnioskowania, Schlussregeln)[16]. Now the difference between a law and a rule seems to be two-fold; (1) while laws are stated in object-language, rules are formulated in meta-language, (2) while in most of the laws (namely those which are conditional formulae) the antecedent implies the consequent, in most rules instead of implication we have *entailment*. In our fragments neither of these two characteristics is explicitly applied to the arguments. Since, however, we know that those arguments are not conditional propositions, it is not improbable that the Stoics meant them to assert entailment, as opposed to implication (any kind of implication: we remember here that the strongest, viz. the Diodorean, was meant.) On the other hand, it is difficult to suppose that the Stoics meant their arguments to be metalogical formulae. Not only we lack a positive ground for thinking so, but we have a set of arguments which were considered by them as "unmethodical" because they contained metalogical formulae. We still know at least four such laws: they were formed by substitution in rules analogous to the following (in which $\ulcorner S\ulcorner p\urcorner\urcorner$ stands for: $\ulcorner$you (or Dion) says that $p\urcorner$):

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[16] Łukasiewicz, *Zur Geschichte 114f.* Another difference is also stressed sometimes, viz. between the normative nature of a rule and a descriptive nature of a law. But this does not seem to be essential; we can always say, instead of $\ulcorner p\urcorner$ must be admitted", which is normative, "$\ulcorner p\urcorner$ belongs to the class a" which is descriptive.

16. 01. $F\ulcorner pq \urcorner . p . \supset . \sim q$ [17]
16. 02. $p . S \ulcorner p \urcorner . \supset .$ what you say is true [18]
16. 03. $S \ulcorner p \urcorner .$ what you say is true$. \supset . p$ [19]
16. 04. $\ulcorner q \urcorner$ follows from $\ulcorner p \urcorner . p . \supset . q .$ [20]

Our interpretation is not quite adequate for, as it must be remembered, the Stoics were dealing with propositions not with sentences, while we formulated the above laws as bearing on sentences. They show nevertheless how sharply those logicians distinguished between a fact and the truth of the sentence which means it — especially 16. 01 as compared with 16. 23. One would only wish that some contemporary logicians were as careful as those old thinkers were.

16 C. The schemes of inference; axiomatization

The Stoic distinguished an argument (*λόγος*) from a mode (*τρόπος*); the latter was considered as an "outline of an argument" [21]. In modes, ordinal numerals ("the first", "the second" etc.) were used as variables. Here is an instance: "if the first, the second; but, the first; consequently (*ἄρα*) the second". Those numerals are clearly propositional variables: this is the second important difference between the Aristotelian and the Stoic logic, as the variables contained in the Aristotelian formulae are nearly always class-name-variables (8 B).

We meet sometimes with mixed forms, half-arguments, half-modes (*λογότροπος*), e.g. "If Plato is alive, then Plato breathes; the first; therefore the second" [22] Modes were divided into valid (*ὑγιές* or *συνακτικός*) and invalid (*φαῦλον* or *μοχθηρόν*) [23].

Those modes were axiomatically arranged. Five of them, namely the "undemonstrated", were assumed as axioms and other modes were reduced to them by means of four general rules (*θέματα*) [24]. Out of those four we still have two, the first and the third. The first was a (meta-)metalogical rule analogous to

[17] *DL 7, 78.* — [18] *Alex. An. Pr. 22, 17ff.* — [19] *Gal. Inst. 42, 18*; *Alex. An. Pr. 345, 28ff.* — [20] *Alex. An. Pr. 373, 31—35.* — [21] *AM 8, 227*; *DL 7, 76*; *Gal. Inst. 15, 8ff.* — [22] *DL 7, 77*; cp. *AM 8, 306.* — [23] *HP B 154*; *AM 8, 429, 444*, cp. *413.* — [24] Galen apud *SVF 248 p. 83, 24—26.*

16\. 11. $pq \supset r . \supset . \sim rq \supset \sim p$

16\. 12. $pq \supset r . \supset . p \sim r \supset \sim q$ [25]

while the third was analogous to

16\. 13. $pq \supset r . s \supset p . \supset . sq \supset r$

16\. 14. $pq \supset r . s \supset q . \supset . ps \supset r .$ [26]

It will be seen that those two are closely similar to the Aristotelian 8. 11 f. and 8. 21 f.; this throws a light on the origin of some of the Stoic doctrines. The two other rules are lost; but a rule, according to which a proposition potentially (*δυνάμει*) contained in the premisses may be added to them — i.e. a rule analogous to

16\. 15. $p . p \supset q . \supset . pq$

(with an indeterminate number of permisses for $\ulcorner p \urcorner$) — is reported by Sextus [27] and might have been one of the two missing "themes".

The Stoics had also another important rule which we shall call after Dr Mates, "the principle of conditionalization". It may be formulated as follows: $\ulcorner\ulcorner\ulcorner q \urcorner$ is deducible from $p \urcorner$ is equivalent to $\ulcorner p \rightarrow q \urcorner\urcorner$ (where Diodorean implication is meant). [28]

16 D. The undemonstrated

As it has already been said, the Stoics had undemonstrated modes as well as undemonstrated arguments; they were considered as axioms of all logic, [29] and certainly played a considerable role in Stoic logic — we find them listed in reports in at least eight different sources (by six or seven authors). [30] The number generally given is five; it seems certain that they were all contained in the writings of Chrysippus [31], but we do not know if he is to be considered as their inventor. Since, however, some sources give more

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[25] *Apul. 277f.*, cp. *278* p. *191, 8—10.* — [26] *Alex. An. Pr. 278, 6ff.*; *Simpl. Cael. 336, 33ff.* — [27] *AM 8, 231.* — [28] *AM 8, 415ff.*; cp. *HP B 113, 137.* — [29] *DL 7, 79.* — [30] *HP B 157ff.*; *AM 8, 223ff.*; *DL 7, 79ff.*; *Gal. Inst. 15ff.*; *Gal. Hist. Phil. 15*; *Philop. An. Pr. 244ff.*; *Amm. An. Pr. 68*; *Mart. 4, 414ff.* — [31] *AM 8, 223*; *DL 7, 79*; *Gal. Inst. 15, 11ff.*; *33, 16ff.*; *34, 24f.*

than five undemonstrated,[32] it seems that there was some discussion about their number. In our sources the undemonstrated are both stated and described. We give here the corresponding logical laws, reminding the reader that in fact they were rules:

* 16. 21.	$p \supset q . p . \supset . q$ [33]
* 16. 22.	$p \supset q . \sim q . \supset . \sim p$ [34]
* 16. 23.	$\sim (pq) . p . \supset . \sim q$
* 16. 24.	$p \vee q . p . \supset . \sim q$ [35]
* 16. 25.	$p \vee q . \sim p . \supset . q$ [35]

A certain difficulty is met in the description of 16. 25: we have there *ἀντικείμενον* while *ἀντιφατικόν* would be expected; on the whole, however, this list is the best known logical theory of the Stoic School.

16 E. Derived modes

In spite of what Cicero says, (that the number of consequences drawn from the undemonstrated was "innumerable")[36] we know only of a class of derived modes whose elements are not only numerable but are even rather few. Here is the list of the logical analoga:

* 16. 31. $p \supset p . p . \supset . p$ [37]

Which is one of the "duplicated" modes already mentioned.

* 16. 32.	$p . \supset . p \supset q : p : \supset : q$ [38]
* 16. 33.	$pq \supset r . \sim r . p . \supset . \sim q$ [39]
* 16. 34.	$p \vee q \vee r . \sim p \sim q . \supset . r$ [40]

This rule, Chrysippus said, is used even by dogs.

* 16. 35. $p \supset q . p \supset \sim q . \supset . \sim p$ [41]

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[32] *Cicero Top. 57*; *Gal. Inst. 32, 19f.* — [33] also: *Amm. An. Pr. XI.* — [34] also: *Alex. Top. 166*; *Boet. De Int. 2, 351.* — [35] also: *Alex. Top. 175.* — [36] *Cicero Top. 57.* About the number of possible molecular propositions cp. *SVF 210* (*Plut. De Stoic. rep. 29*). — [37] *Alex. An. Pr. 20, 10.* — [38] *AM 8, 230—233.* — [39] *AM 8, 234—41.* — [40] *HP A 69.* — [41] *Origen. C. Celsum 7, 15* p. *166f.*

The name of the rule 16. 35 was "from two molecular premisses" (*διὰ δύο τροπικῶν*).

* 16. 36. $p \supset p . \sim p \supset . p . p \vee \sim p . \supset . p.$[42]

It is not only possible, but even probable, that more such derived modes will be found once the great material contained in the writings of the Commentators has been elaborated by competent logicians.

Here is an instance of the reduction of such arguments to the undemonstrated, that of 16. 33.[39] It is stated in the following form:

(1) If both the first and the second, then the third;
(2) Not the third
(3) The first
(4) Therefore not the second

From (1) and (2) we obtain by 16. 22

(5) Not both the first and the second.

(5) is now considered as a new premiss (16. 15!) and thus, combining (3) and (5) we obtain by 16. 23 the conclusion (4) which was to be demonstrated. 16. 33 has been "analyzed" into 16. 22 and 16. 23.

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[42] *AM 8, 281, 466* and *HP B 131f.*; *Amm. An. Pr. XI.*

17. Invalid arguments and paradoxes. The Liar

Already the first Megaricians were very interested in sophistics; this remained true for the whole of the school. Among many paradoxes that the Megaricians proposed only one has a real importance — and this a great one — that of the Liar. We shall sketch here first the Stoic classification of invalid arguments (A), then, after a short mention of different paradoxes (B), we shall describe a few things we still know about the Liar (C).

17 A. Invalid arguments

According to Sextus, the Stoics distinguished four classes of invalid arguments (*ἀσύνακτοι* [1] or *ἀπέραντοι* [2]). They were the following: (1) incoherent (*παρὰ διάρτησιν* [3]), where there is no logical connection between the premisses or between the premisses and the conclusion; (2) redundant (*παρὰ παρολκήν* [4]) with a useless premiss; the elements of this class are, in fact, quite valid; (3) formed out of an invalid scheme (*ἐν μοχθηρῷ σχήματι* [5]) e.g. out of $\ulcorner p \supset q . \sim p . \supset . \sim q \urcorner$; this is the class studied by Aristotle under the name of "ignoratio elenchi" [6] — and it contains (as Aristotle had noticed already) all invalid arguments; (4) deficient (*παρὰ ἔλλειψιν* or *παρὰ παράλειψιν* [7]), where the disjunction in the premiss is incomplete. From other texts we know that they also studied the vicious-circle fallacy (*ὁ διάλληλος τρόπος*) [8].

17 B. The paradoxes

Eubulides of Miletus is credited [9] with the invention of four paradoxes, namely the Liar, the swindler, the concealed (*ἐγκεκαλυμμένος*), the baldhead, the heap (*σωρίτης*: one grain does not make a heap, nor two etc. . . .), the Electra (who knows and does not know his veiled brother) and the horned (*κερατίνης*: you have what you did not lose; consequently you have horns). This set has been increased considerably by the Stoics [10]; among those

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[1] *HP B 146ff.*; *152f.* — [2] *AM 8, 429ff.* — [3] *HP B 146*; *AM 8, 430.* — [4] *HP B 147*; *AM 8, 431.* — [5] *HP B, 147*; *AM 8, 432f.* — [6] cp. ch. 6 D. — [7] *HP B 150*; *AM 8, 434*; cp. *Gellius 2, 7*; *5, 11.* — [8] *HP B 114.* — [9] *DL 2, 108.* — [10] Full descriptions in *Prantl 42ff.* and *487ff.*

additions we may mention the "answering" (*ἀποφάσκον*: man is not Socrates, Socrates is man; thus Socrates is not Socrates); the "nobody" (*οὔτις*: who is in Athens is not in Megara; a man is in Athens; thus there is no man in Megara); the crocodile (*ἄπορος* or *κροκοδειλίτης* also called *ἀντιστρέφον*: the crocodile took the baby of a woman and said he will give it back if she answers rightly his question. The question is: will I give the child back? The woman says no; then both parties are able to deduce the conclusion they desire: the crocodile, because if the answer was right, he cannot restore the child according to the truth and if it was wrong, according to the convention; the woman for opposite reasons.) It will easily be seen that those paradoxes are really without any logical interest. Historically they were important in as much as both during the ancient times and the Middle Ages many logicians wrote ample studies on them — and these puzzles gave occasion to several relevant doctrines. Only one ancient paradox — if we except the heap, which is connected with the problem of the continuum, is still of importance — this is the Liar.

17 C. The Liar

This paradox (*ὁ ψευδόμενος*) was not yet known to Plato (at about 387 B.C.[11]) and is already quoted by Aristotle in the *Soph. El.*[12] i.e. about 330 B.C.; consequently it is not improbable that it might be the work of Eubulides to whom it is ascribed by Diogenes.[13] Later on we have different forms of it stated by Cicero[14], St. Paul[15], Gellius[16], Lucian[17], St. Hieronymus[18], and many later writers[19]. We know also that Theophrastus wrote three books on the Liar[20], while the list of the works of Chrysippus contain at least six titles of works (with fourteen books) dealing with it.[21] Moreover, we know that at least one Greek logician died

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[11] cp. Rüstow *43*. — [12] *Soph. El. 25, 180 b 2—7*; *Eth. Nic. H 3, 1146 a 21—27*. The last test is dubious, however, cp. Rüstow *53*. — [13] *DL 2, 108*. — [14] *Ac. Pr. II 95, 96*. — [15] *Tit. 1, 12—13*. — [16] *Gell. 18, 2*. — [17] *Ver. Hist. 1, 4*. — [18] *Epist. LXIX ad Oceanum, 2*. — [19] Enumeration in Rüstow *40f*. — [20] *DL 5, 49*; *Usener 59, 11*. — [21] *DL 7, 189ff*. (*σ. V, 1—3*; *VI, 2*; *VII, 1*; *VI, 6*) cp. Rüstow *63ff*.

because of the Liar; this was a certain Philitas of Kos (340—285) whose epitaph, stating that the Liar "killed him", is preserved by Athenaeus [22]. Unfortunately we do not have a unique form of the paradox; the quotations of it may be grouped in three classes. Cicero and Gellius have a simple question: "if you truly say that you lie, are you lying?"; St. Paul states that a Cretan said that all Cretans always lie; in another text of Cicero we have: "if you say that you lie and you say it truly, you lie"; but some later authors conclude the opposite: "if I say that I lie and I lie, I say the truth"[23], while others still conclude that the sentence is true if it is false and conversely. [24] Rüstow thought [25] that all those forms may be combined in a single dialogue — but this is by no means certain.

We know still less about the attempts of solution. The only text we still have is that of Aristotle. He classes the Liar with the second kind of fallacies independent of speech, namely those which are called "use of the words with or without qualification"; he says that the sentence in question may be, absolutely speaking, true, but false in some respects [26] — which does not touch the real problem. A. Rüstow advanced a hypothesis about Theophrastus' [27] teaching and another about the solution of Chrysippus. [28] The first seems quite unsubstantiated, however, while the latter — according to which Chrysippus declared the sentence to be meaningless — does not seem sufficiently proved; the text on which it is based is very short and enigmatic. [29]

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[22] *Athaen. 9, 401 E: Ξεῖνε, Φιλητᾶς εἰμί. λόγων ὁ ψευδόμενός με — ὤλεσε, καὶ νυκτῶν φροντίδες ἑσπέριοι.* — [23] *Lucian l. cit.; Ps. Acro ad Hor. ep. II, 1, 47; Placidus, Goetz Corp. Gloss 153.* — [24] *August., C. Acad. 3, 29.* — [25] Rüstow *40f.* — [26] *Soph. El. 25, 180 b 5ff.* — [27] Rüstow *54.* — [28] *ib. 84ff.* — [29] *ib. 74. 16—18,* cp. Rüstow's commentary *p. 80.*

VI. THE LAST PERIOD

18. Greek Logic after Chrysippus. Boethius

In spite of the fact that we possess a quantity of good texts for this period we have hardly any studies on the logicians belonging to that time; in those conditions it will be preferable to limit ourselves to a general sketch of the situation (A), to the enumeration of the main authors (B), and to some remarks about Galenus (C) and Boethius who is a good representative of the last ancient logicians and, at the same time, the main intermediary between Antiquity and the Middle Ages (D—E).

18 A. General survey

The last period of ancient logic is characterized by the following traits, some of which have already been touched upon (ch. 2 C). First of all, as far as we know, it is no longer a creative period: we cannot quote a single logician comparable — not only with Aristotle, Diodorus or Chrysippus, but even with Theophrastus. Logic seems to have still been much studied, however, and its knowledge must have been widely spread. At the same time there was the unfortunate phenomenon of the struggle between the Peripatetic and the Stoic Schools. Slowly a mixture of both trends formed. Thus, we hear that Boethus of Sidon, pupil of Andronicus Rhodos, who lived at the time of Augustus and was the head of the Peripatetic School, asserted the priority of the Stoic undemonstrated in regard to the categorical syllogism; syncretism is often met with later on, e.g. in the Dialectical Introduction of Galenus. On the other hand there are still some rigid peripateticians who deny any merit to the Stoic-Megaric School; Alexander of Aphrodisias is an instance. In the long run, however, a kind of commonly received doctrine, composed of rather poor remains of both Aristotelian and Stoic-Megaric doctrines was formed. Yet the work of the commentators and authors of textbooks has not been, as it seems, completely irrelevant to logic — here and there they probably were able to bring some complements and perfections of the old doctrines. Unfortunately, we know nearly nothing about their work.

18 B. The logicians

There follows here a (incomplete) list of important logicians who lived during that long period. Ariston of Alexandria is reported to have stated the "subaltern modes" of the syllogism [1]; he lived during the II century A.D. Another important logician of the same period is the famous physician Galenus (129 — c. 199 A.D.); his "Dialectical Introduction" is the only ancient Greek textbook of logic preserved; it has been studied by Fr. Stakelum. His contemporary Apuleius of Madaura (125 A.D.) wrote among others a latin book *περὶ ἑρμηνείας* which seems to be of great interest. Alexander of Aphrodisias, who lived during the third century, is probably one of the most penetrating logicians of the peripatetic School and one of the best commentators of the Organon in history. Porphyrius of Thyrus (232/3 — beginning of the IV century) is another important commentator of Aristotle, if inferior to Alexander: his Introduction was destined to have a brilliant career during the Middle Ages. Sextus Empiricus (3rd century) our main source for the Stoic-Megaric School can hardly be called a logician, yet he knew logic well and some of his criticisms might be of interest. Later authors — such as Iamblichus of Chalkis († c. 330), Themistius (330—390), Ammonius Hermeiou, the disciple of Proclus, David Ioannes Philoponus († after 640), are of far lesser importance. But at the end of our period we have again some men of interest: Martianus Capella, who wrote between 410 and 439 his celebrated "De nuptiis Philosophiae et Mercurii" with a book devoted to logic; Simplicius, pupil of Ammonius, and the last important Athenian Philosopher (he was driven from Athens by a decree of Justinian in 529) is also an intelligent logician; finally Boethius, himself a not very good thinker, is highly important because of his influence on the Middle Ages, but also because of the mass of information his logical works contain.

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[1] *Apul. 193, 16ff.*; there is much confusion in this text.

18 C. Galenus

The logic of Galenus has not yet been fully studied, but following a hint made by Łukasiewicz [2] Stakelum examined the Dialectical Introduction [3] and the results of his inquiry show that we have to do with a highly interesting logical work. The most original theory contained in that booklet is perhaps the division of all syllogisms into three classes: the hypothetical (ὑποθετικοί) which are the Stoic modes [4]; categorical (κατηγορικοί) [5] which are the Aristotelian syllogisms stated in a purely metalogical and Stoic form, with the laws 9. 67—69 [6] and 9. 72 [7] (but with explicit rejection of the fourth "Galenic" figure [8]) and, finally, the relative syllogisms (κατὰ τὸ πρός τι) [9]. As instance of the last class several mathematical laws are given, such as:

$$a = b . c = d . \supset . a + c = b + d$$ [10]

or

$$x = z . y = z . \supset . x = y$$ [11]

and the practice of mathematicians is quoted [12] — a rather exceptional thing in post-Aristotelian logic. Later on, Galenus says that all such laws can be analyzed into hypothetical syllogisms "concerning numbers or other things which belong to the category of relation [13]" and supplies (using a substitution) the following scheme:

18. 01. $$xRy \supset y\breve{R}x . aRb . \supset . b\breve{R}a.$$ [14]

The Aristotelian formulae 11. 56 ff. are said to belong to the same class. [15]

It is easy to see that the results of Galenus' research into the logic of relations are rather poor; still he divided logic exactly as the authors of the *Principia* did and asserted the existence of special laws concerning relations. [16]

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[2] Łukasiewicz, *Geschichte*. — [3] *Gal. Inst.* — [4] *15ff.* — [5] *17ff.* — [6] *25, 9—13.* — [7] *26, 5f.*; reading πρῶτον *26, 6.* — [8] *26, 14f.* — [9] *38ff.* — [10] *39, 7—10.* — [11] *39, 19.* — [12] *39, 20ff.* — [13] *39, 19.* — [14] *41, 11—13.* — [15] *41, 16ff.* — [16] *38, 12ff.* The division is due to the Stakelum.

18 D. Boethius: generalities

The Roman statesman and philosopher Manlius Severinus Boethius (* c. 480 † 524/5), the last Roman philosopher, was influenced in logic by a Roman Grammarian Marius Victorinus (IVth century), but seems to depend above all on Greek sources. He wrote a number of books on logic, among which the *De Interpretatione* (in two editions), the *De Syllogismo Hypothetico* and the *De Syllogismo Categorico* are the most important. We meet in those works with a complete Latin terminology for logic and also with an arrangement of many Aristotelian doctrines which will be from his time on accepted as the "classical" one. Thus the syllogisms are stated not in form of conditional propositions, but as the Stoic λόγοι, in the form of rules: $\ulcorner p$; q; igitur $r\urcorner$ he always writes. He also has the complete "logical square" and many other details of the "classic" logic. All his work would merit a thorough examination.

Actually the only part of his logic on which we have some information, thanks to the recent works of Dürr and R. van den Driessche, is the theory of "hypothetical" syllogisms. It is probably copied from a Greek author. The status of the variables is not clear in their laws; as they are stated they might equally well be class or propositional variables, as it was already the case with Theophrastus. But there is in Boethius' logic an important innovation which obliges us to take them for propositional variables: he constantly substitutes propositional functions for variables. At the same time he states (it is true in a rather vague form (8.47 C)) the law of double negation

18. 02. $$\sim \sim p \equiv p$$

and uses also, without stating it, although consciously, the law of negation of the implication:

[18. 03.] $$\sim (p \supset q) \equiv p \sim q.$$

Finally, he uses sometimes, as van den Driessche has shown, the "si" for "if and only if", i.e. as standing for our "$\equiv$".

18 E. Boethius: formal laws of hypothetical syllogism

There are first of all eight "simple" laws formed by substitution in 16. 21—22:

* 18. 11. $p \supset q . p . \supset . q$ [17]
* 18. 111. $p \supset \sim q . p . \supset . \sim q$ [18]
* 18. 112. $\sim p \supset q . \sim p . \supset . q$ [19]
* 18. 113. $\sim p \supset \sim q . \sim p . \supset . \sim q$ [20]
* 18. 12. $p \supset q . \sim q . \supset . \sim p$ [21]
* 18. 121. $p \supset \sim q . q . \supset . \sim p$ [22]
* 18. 122. $\sim p \supset q . \sim q . \supset . p$ [23]
* 18. 123. $\sim p \supset \sim q . q . \supset . p.$ [24]

Similar substitutions are found in all following formulae; we shall omit them however and give only the main laws. We have, first, four such laws in which the "si" means "if and only if":

* 18. 21. $p \equiv q . p . \supset . q$ [25]
* 18. 22. $p \equiv q . \sim p . \supset . \sim q$ [26]
* 18. 23. $p \equiv q . \sim q . \supset . \sim p$ [27]
* 18. 24. $p \equiv q . q . \supset . p.$ [28]

The next class is that of theorems similar to the Theophrastian "totally hypothetical" syllogisms (12. 31 ff.):

* 18. 31. $p \supset q . q \supset r . p . \supset . r$ [29]
* 18. 32. $p \supset q . q \supset r . \sim r . \supset . \sim p$ [30]
* 18. 33. $p \supset q . \sim p \supset r . \supset . \sim q \supset r$ [31]
* 18. 34. $p \supset q . \sim p \supset r . \supset . \sim r \supset q$ [32]
* 18. 35. $q \supset p . r \supset \sim p . \supset . q \supset \sim r$ [33]
* 18. 36. $q \supset p . r \supset \sim p . \supset . r \supset \sim q$ [34]

Then we have 16. 24—25 with the addition of the two following:

* 18. 37. $p \vee q . q . \supset . \sim p$ [35]
* 18. 38. $p \vee q . \sim q . \supset . p$ [36]

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[17] *845 B.* — [18] *845 C.* — [19] *845 D.* — [20] *846 B.* — [21] *846 D.* — [22] *847 B.* — [23] *847 D.* — [24] *848 B.* — [25] *845 D*; Here and in the following we give the interpretation of van den Driessche. — [26] *848 A.* — [27] *847 A.* — [28] *846 A.* — [29] *856 B.* — [30] *858 B.* — [31] *859 D.* — [12] *860 B.* — [33] *864 B.* — [34] *874 D.* — [35] *874 D.* — [36] *ib.*

and finally two laws with the non-exclusive alternative:

* 18. 39.	$p \vee q . \sim p . \supset . q$ [37]
* 18. 40.	$p \vee q . \sim q . \supset . p$ [38]

Those are the basic theorems; out of them Boethius forms, by substituting propositional functions for variables, several derived laws, in the deduction of which 18. 03 is also sometimes used. From 18. 11 he obtains:

* 18. 41.	$p . \supset . q \supset r : p : \supset : q \supset r$ [39]
* 18. 42.	$p \supset q . \supset . r : p \supset q : \supset : r$ [40]
* 18. 43.	$p \supset q . \supset . r \supset s : p \supset q : \supset : r \supset s.$ [41]

From 18. 12 he deduces:

* 18. 51.	$p . \supset . q \supset r : q \sim r : \supset : \sim p$ [42]
* 18. 52.	$p \supset q . \supset . r : \sim r : \supset : p \sim q$ [43]
* 18. 53.	$p \supset q . \supset . r \supset s : r \sim s : \supset : p \sim q.$ [44]

From 18. 21:

* 18. 61.	$p . \equiv . q \supset r : p : \supset : q \supset r$ [45]
* 18. 62.	$p \equiv q . \supset . r : p \equiv q : \supset : r$ [46]
* 18. 63.	$p \supset q . \equiv . r \supset s : p \supset q : \supset : r \supset s.$ [47]

From 18. 22:

* 18. 71.	$p . \equiv . q \supset r : \sim p : \supset : q \sim r$ [48]
* 18. 72.	$p \supset q . \equiv . r : p \sim q : \supset : \sim r$ [49]
* 18. 73.	$p \supset q . \equiv . r \supset s : p \sim q : \supset : r \sim s.$ [50]

From 18. 23:

* 18. 81.	$p . \equiv . q \supset r : q \sim r : \supset : \sim p$ [51]
* 18. 82.	$p \supset q . \equiv . r : \sim r : \supset : p \sim q$ [52]
* 18. 83.	$p \supset q . \equiv . r \supset s : r \sim s : \supset : p \sim q$ [53]

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[37] *875 A, 876 B.* — [38] *ib.* — [39] *851 C.* — [40] *854 B.* — [41] *872 A.* — [42] *851 C.* — [43] *854 B.* — [44] *872 A.* — [45] *852 A—B.* — [46] *854 C—D.* — [47] *872 B—C; 873 A—B.* — [48] *852 A—B.* — [49] *854 C—D.* — [50] *872 B—C; 873 A—B.* — [51] *852 A—B.* — [52] *854 C—D.* — [53] *872 B—C; 873 A—B.*

From 18. 24:

*18. 91.	$p . \equiv . q \supset r : q \supset r : \supset : p$ [54]
*18. 92.	$p \supset q . \equiv . r : r : \supset : p \supset q$ [55]
*18. 93.	$p \supset q . \equiv . r \supset s : r \supset s : \supset : p \supset q .$ [56]

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[54] *852 A—B.* — [55] *854 C—D.* — [56] *872 B—C; 873 A—B; 874 A.*

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List of abbreviations

Alex. An. Pr. 1. — *Alex. Met.* 3. — *Alex. Top.* 2. — *AM* 54, 55. — *Amm. An. Pr.* 4. — *Amm. De Int.* 5. — *An. Post.* 11. — *An. Pr.* 11. — *Apul.* (*De Int.*) 7. — *Athaen.* 21. — *August.* 22. — *Bekker Anecd.* 24. — *Boeth. De Int.* 28. — *Boeth. De Syll. Cat.* 25. — *Cat.* 10. — *Cic. Acad.* 33. — *Cic. De fato* 34. — *Cic. Top.* 30. — *Cic. Tusc.* 31. — *Crat.* 50. — *D.* 35. — *De An.* 14. — *De Int.* 10. — *De part. an.* 8. — *De somn. et vig.* 8. — *Diels* 35. — *DL* 37. — *Eth. Nic.* 16. — *Epict.* (*Diss.*) 38. — *Epist. LXIX ad Oceanum* 42. — *Euth.* 50. — *Gal. Hist. Phil.* 40. — *Gal. Inst.* 41. — *Gell*[*ius*] 23. — *Gal. De Soph.* 40. — *Gorg.* 50. — *HP* 55. — *Mart. Capella* 45. — *Meno* 50. — *Met.* 15. — *Origen. C. Cels.* 46. — *Philo* 47. — *Philop. An. Pr.* 48. — *Phys.* 13. — *Placidus* 43. — *Plot.* 51. — *Plut. De Stoic. rep.* 52. — *Poet.* 18. — *Prot.* 50. — *Ps. Acro* 53. — *Rhet.* 17. — *Rose* 19. — *Simpl. Cael.* 56. — *Simpl. Cat.* 57. — *Simpl. Phys.* 58, 59. — *Soph.* 50. — *Soph. El.* 12. — *SVF* 60. — *Theaeth.* 50. — *Tim.* 50. — *Top.* 12. — *Ver. Hist.* 44. — *Waitz* 9.

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[1]) Only a selection of works is quoted. The following have been omitted (with few exceptions): (*a*) works written from an older point of view (for Bibliography cp. PHILIPPE, PRAECHTER and VIRIEUX—REYMOND, *La logique*), (*b*) systematic papers on Aristotle's syllogistic (Bibliography in *The Journal of Symbolic Logic* 1936ff. cp. Beth., *Symbolische Logik*). Cp. also the remarks on ancient and medieval history of logic p. 4.

ADDITIONS

Ch. 1 A, note: During the printing of the present book, a new work on Indian Logic, written from the viewpoint of Mathematical Logic, came to the Author's knowledge: D. H. H. INGALLS, *Materials for the Study of Navya-Nyaya Logic*, Cambridge, Mass. 1951.

Ch. 9 ff. Throughout the chapters referring to Aristotle's doctrines the reader should keep in mind what was said about the existential import of sentences, p. 44; without this assumption many Aristotelian laws are, of course, invalid.

Ch. 10 ff. The author had the exceptional privilege of reading in manuscript the work of Professor ŁUKASIEWICZ on Aristotle's syllogistics. Professor ŁUKASIEWICZ finds that Aristotle had no clear idea about what he called "syllogisms based on hypothesis." Cf., however, the remarks in the Author's paper in Methodos (this contains also all texts concerned in full).

Ch. 13 ff. The Author's indebtedness to Dr MATES is perhaps greater than would appear from his statement in the Preface. As a matter of fact, in the chapters concerning the Stoics, Dr MATES' work was followed with very few additions and changes. This, however, makes Dr MATES in no way responsible for any misstatements which may be found in the present account.

Ch. 16 A, Note 11. In the text, ŁUKASIEWICZ's point of view has been followed. Dr MATES, however, thinks (with several arguments to support him) that some of the *ἀναπόδεικτα* were demonstrable.

Ch. 18 D f. In his recent paper Father THOMAS has shown that there is much confusion in BOETHIUS' logical thought; a point in question is, among others, N. 18.02.

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